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Brief paper

Iterative learning control for large scale nonlinear systems with observation noise[☆]

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ABSTRACT

The iterative learning control (ILC) is constructed for the discrete-time large scale systems. Each subsystem is affine nonlinear and its observation equation is with noise. Subsystems are nonlinearly connected via the large state vector of the whole system. The possibility of data missing, and communication delay is taken into account. It is proved that ILC given in the paper with probability one converges to the optimal one minimizing the tracking error. The simulation results are consistent with theoretical analysis.

1. Introduction

ILC is normally applied to systems which repeatedly perform some tracking tasks and where the input–output information contained in past cycles can be used to improve tracking performance of the next cycle. By large scale system we mean that the whole system is composed of many subsystems which are connected via the large state vector of the system but each subsystem is controlled on the basis of its own input and output information. The petrochemistry processes, power systems, network/networked control systems etc. are typical examples of large scale systems.

There exist some, yet not too many, papers on ILC of large scale systems. In Hwang, Kim, and Bien (1993) the D-type ILC is constructed for the deterministic continuous-time linear systems, where by D-type ILC it is meant that the differential of the tracking error is fed to the systems. In Wu, Kawabata, and Kawabata (2003), and Wu (2007) similar results are obtained for both the time-invariant and time-varying linear large scale systems with no control in the observation equation. In Ruan, Bien, and Park (2008) a PID type control updating algorithm is proposed to follow non-repeated target trajectories for the time-invariant continuous-time deterministic linear large scale systems, and the algorithm is shown to be bounded in the $\mathcal{L}_p$ norm sense. In Ruan et al. (2008) the results are obtained for SISO subsystems, and they are extended to MIMO subsystems in Ruan, Wu, Li, and Wan (2009), and to the affine nonlinear subsystems in Ruan, Chen, and Wan (2008). In all these works the systems are of continuous time, the subsystems are linearly connected via state vector, and no noise is considered.

In the random environment, ILC is considered for linear stochastic systems (Chen, 2003) and for the stochastic affine nonlinear systems in Saab (2005), Chen and Fang (2004) and convergence of ILC is established. In contrast to Saab (2005), where no state noise is considered, both state and observation noises are taken into account in Chen and Fang (2004).

We consider the discrete-time large scale systems with nonlinearly connected subsystems, each of which is affine nonlinear, and the observation equations are with noises. The possibility of data missing, communication delay, and asynchronous control updating are taken into account. The asynchronous distributed ILC is designed based on asynchronous stochastic approximation, and the control sequence is proved to converge with probability one to the optimal one minimizing the tracking error.

The rest of the paper is arranged as follows. The problem is formulated in Section 2, and the optimal control is shown in Section 3. The ILC algorithms and their convergence analysis are given in Section 4. A numerical example is demonstrated in Section 5. Some concluding remarks are given in Section 6.

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2. Problem formulation

Let the large scale system be composed of n subsystems, and let its ith subsystem be described by

$$\begin{cases} x_i(t+1,k) = f_i(t, x(t,k)) + b_i(t, x(t,k))u_i(t,k), \\ y_i(t,k) = c_i(t)x_i(t,k) + w_i(t,k), \end{cases} \tag{1}$$

where $t \in \{0, 1, \ldots, T\}$ denotes the time instant in a cycle of the process, while $k = 1, 2, \ldots$ label different cycles, $u_i(t,k) \in \mathbb{R}$, $x_i(t,k) \in \mathbb{R}^{n_i}$, and $y_i(t,k) \in \mathbb{R}$ denote the input, state, and output of the ith subsystem, respectively. Denote by $x(t,k) = [x_1^T(t,k), \ldots, x_n^T(t,k)]^T$ the state vector of the large scale system. The subsystems are connected via the vector functions $f_i(t, x(t,k))$, which, however, together with $c_i(t)$ and $b_i(t, x(t,k))$ are unknown. For the ith subsystem the tracking target is a given signal $y_i(t,d)$, $t \in \{0, 1, \ldots, T\}$.

Let $\mathcal{F}_{i,k} \triangleq \sigma\{y_i(t,j), x(t,j), w_i(t,j), 0 \le j \le k, t \in \{0, 1, \ldots, T\}\}$ be the σ-algebra generated by $y_i(t,j), x(t,j), w_i(t,j), 0 \le j \le k, t \in \{0, 1, \ldots, T\}$, and let $\mathcal{F}_{i,k}^y \triangleq \sigma\{y_i(t,j), 0 \le j \le k, t \in \{0, 1, \ldots, T\}\}$ be the σ-algebra generated by $y_i(t,j)$, $0 \le j \le k$, $t \in \{0, 1, \ldots, T\}$. For the ith subsystem the set of admissible controls is defined as

$$U_i = \{u_i(t, k+1) \in \mathcal{F}_{i,k}^y, \sup_k \|u_i(t,k)\| < \infty, \text{ a.s.}$$
$$t \in \{0, 1, \ldots, T-1\}, \ k = 0, 1, 2, \ldots\}.$$

It is required to find the control sequences $\{u_i(t,k), k = 0, 1, 2, \ldots\} \in U_i$, $\forall i = 1, \ldots, n$ to minimize the following tracking errors $\forall t \in \{0, 1, \ldots, T\}$

$$V_i(t) = \limsup_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \|y_i(t,d) - y_i(t,k)\|^2, \tag{2}$$

The desired control sequence is to be recursively defined in Section 4 by the algorithm (8). For the ith subsystem (1), $\forall i \in \{1, \ldots, n\}$, the following assumptions are used.

A1. The target signal $y_i(t,d)$ is realizable in the sense that there exist $u_i(t,d)$ and $x_i(0,d)$ such that

$$\begin{cases} x_i(t+1,d) = f_i(t, x(t,d)) + b_i(t, x(t,d))u_i(t,d), \\ y_i(t,d) = c_i(t)x_i(t,d), \end{cases} \tag{3}$$

where $x(t,d)$ is defined similarly to $x(t,k)$.

A2. The initial values can asymptotically be precisely reset in the sense that $x_i(0,k) - x_i(0,d) \xrightarrow[k\to\infty]{} 0$.

In ILC how to take the initial values $x(0,k)$, $k = 1, 2, \ldots$ is called the re-initialization problem, which has been studied in many papers, e.g., Chen, Wen, Gong, and Sun (1999), and Xu, Yan, and Chen (2006) among others. In A2, $x(0,d)$ is assumed to be asymptotically available. In some papers, people simply assume $x(0,d)$ is available (Chien & Tayebi, 2008; Xu, Xu, & Lee, 2005; Xu et al., 2006), or treat $x(0,k)$ as a random variable with mean equal to $x(0,d)$ (Chen, 2003; Chen & Fang, 2004).

A3. The functions $f_i(\cdot,\cdot)$ and $b_i(\cdot,\cdot)$ are continuous with respect to the second argument.

From the proof of Lemma 1 and Theorem 2 it will be seen that the functions $f_i(t,x)$ and $b_i(t,x)$ are allowed to have discontinuities with respect to x, but are required be continuous at $x = x(t,d)$. Since $x = x(t,d)$ is unknown, we simply assume A3.

A4. The real number $c_i(t+1)b_i(t,x)$ coupling the input and output is an unknown nonzero constant, but its sign, characterizing the control direction, is assumed known.

Without loss of generality, we may assume that $c_i(t+1)b_i(t,x) > 0$. It is noted that $c_i(t+1)b_i(t,x) \neq 0$ is equivalent to claiming that the input/output relative degree is one.

A5. For each t the observation noise $\{w_i(t,k)\}$ is a sequence of independent and identically distributed (iid) random variables with $\mathbb{E}w_i(t,k) = 0$, $\sup_k \mathbb{E}w_i^2(t,k) < \infty$ and $\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^n w_i^2(t,k) = R_i^t$ a.s. $\forall t \in \{0, 1, \ldots, T\}$, where R_i^t is unknown.

It is worth noting that for a large scale system at k the controls $u_i(t,k)$ may not be updated for all $i \in \{1, \ldots, n\}$, because data may be missing and operating rates for subsystems may not be the same.

For simplicity of writing let us set

$$\begin{aligned}
&f_i(t,k) \triangleq f_i(t, x(t,k)), && f_i(t,d) \triangleq f_i(t, x(t,d)),\\
&b_i(t,k) \triangleq b_i(t, x(t,k)), && b_i(t,d) \triangleq b_i(t, x(t,d)),\\
&e_i(t,k) \triangleq y_i(t,d) - y_i(t,k), &&\\
&\delta x_i(t,k) \triangleq x_i(t,d) - x_i(t,k), && \delta u_i(t,k) \triangleq u_i(t,d) - u_i(t,k),\\
&\delta f_i(t,k) \triangleq f_i(t,d) - f_i(t,k), && \delta b_i(t,k) \triangleq b_i(t,d) - b_i(t,k),\\
&c^+ f_i(t,k) \triangleq c(t+1)f_i(t,k), && c^+ b_i(t,k) \triangleq c(t+1)b_i(t,k).
\end{aligned}$$

3. Optimal control

In this section we show the minimum of the index defined by (2) and the optimal control sequence as well.

Lemma 1. *Assume* A1–A3 *hold for System* (1). *If* $\lim_{k\to\infty} \delta u_i(s,k) = 0$, $s = 0, 1, \ldots, t$ $\forall i = 1, 2, \ldots, n$, *then at time* $t+1$, $\|\delta x_i(t+1,k)\| \xrightarrow[k\to\infty]{} 0$, $\|\delta f_i(t+1,k)\| \xrightarrow[k\to\infty]{} 0$, $\|\delta b_i(t+1,k)\| \xrightarrow[k\to\infty]{} 0$, $\forall i$.

Proof. We prove the lemma by induction. From (1) and (3) it follows that for each i

$$\begin{aligned}
&\delta x_i(t+1,k)\\
&\quad = f_i(t,d) - f_i(t,k) + b_i(t,d)u_i(t,d) - b_i(t,k)u_i(t,k)\\
&\quad = \delta f_i(t,k) + \delta b_i(t,k)u_i(t,d) + b_i(t,k)\delta u_i(t,k).
\end{aligned} \tag{4}$$

For $t = 0$, by A2 and A3 we have $\|f_i(0,d) - f_i(0,k)\| \xrightarrow[k\to\infty]{} 0$, $\|b_i(0,d) - b_i(0,k)\| \xrightarrow[k\to\infty]{} 0$, which imply that the first two terms at the right-hand side of (4) tend to zero as $k \to \infty$. Since

$$\|b_i(0,k)\| \le \|b_i(t,d)\| + \|\delta b_i(0,k)\|,$$

it follows that $b_i(0,k)$ is bounded. This incorporating with $\lim_{k\to\infty} \delta u_i(0,k) = 0$ yields that the third term on the right-hand side of (4) also tends to zero as $k \to \infty$, and hence $\delta x_i(1,k) \xrightarrow[k\to\infty]{} 0$, $\forall i$. Further, by A3 it follows that $\|\delta f_i(1,k)\| \xrightarrow[k\to\infty]{} 0$ and $\|\delta b_i(1,k)\| \xrightarrow[k\to\infty]{} 0$, and hence the conclusions of the lemma are valid for $t = 0$.

Assume conclusions of the lemma are true for $t-1$. By the same argument as that used above we find that the conclusions are also valid for t. The lemma is proved. □

Theorem 1. *Assume* A1–A5 *hold for system* (1) *and criterion* (2). *Then*

$$V_i(t) \ge R_i^t, \quad a.s.\ \forall i, t$$

for any admissible control sequence $\{u_i(t,k), i = 1, \ldots, n, t \in \{0, 1, \ldots, T\}\}$. *The inequality becomes equality if* $\delta u_i(t,k) \xrightarrow[k\to\infty]{} 0$, *and in this case* $\{u_i(t,k)\}$ *is the optimal control sequence.*

The theorem can be proved similar to that for Theorem 1 in Chen (2003).

4. Optimal ILC control

Let us denote by $Y_k \subset \{1, 2, \ldots, n\}$ the set of those subsystems which are updated at the kth iteration and denote by $v(i,k)$ the number of control updates occurred up-to and including the kth iteration in system i: $v(i,k)=\sum_{m=1}^{k} I\{i\in Y_m\}$, where $I\{A\}$ is the indicator of the random set A and equals either 1 or zero depending upon whether the event A is true or not. Although the control for a subsystem is not necessary to be updated for each k, it is still required be updated quite frequently. We need the following condition.

A6. There is a number K such that

$$v(i,k+K)-\ v(i,k)>0 \quad \forall i=1,2,\ldots,n,\ \forall k=1,2,\ldots. \tag{5}$$

This condition means that control in any subsystem should be updated at least once during successive K updates of the whole system.

We now define the asynchronous distributed ILC leading the performance index (2) to its minimum. For the ith subsystem the control $u_i(t,k+1)$ at iteration $k+1$ is defined as follows:

$$u_i(t,k+1)=u_i(t,k)+a(v(i,k))I\{i\in Y_k\}e_i(t+1,k), \tag{6}$$

where $\{a(k)\}$ is the step-size such that $a(k)>0$, $\sum_{k=0}^{\infty}a(k)=\infty$, $\sum_{k=0}^{\infty}a(k)^2<\infty$, and $a(j)=a(k)\big(1+O(a(k))\big)$, $\forall j=k-K+1,\ldots,k-1,k$ as $k\to\infty$.

It is clear that $a(k)=\frac{1}{k+1}$ meets these requirements.

Remark 1. The set Y_k is random and it characterizes the asynchronous nature of updating. During the period, where the whole system has been updated k times, the ith subsystem has actually been updated only $v(i,k)\le k$ times.

The algorithm (6) is the asynchronous SA (ASA), which was studied in Borkar (1998), Kushner and Yin (1987), Tsitsiklis (1994), Fang and Chen (2001) and references therein. The ILC based on SA is considered in Chen (2003), Chen and Fang (2004), which, however, concern the synchronous and centralized control.

For any fixed t from (6) we have

$$\delta u_i(t,k+1)=\delta u_i(t,k)-a(v(i,k))I\{i\in Y_k\}$$
$$\cdot[c^+b_i(t,k)\delta u_i(t,k)+\varphi_i(t,k)-w_i(t+1,k)], \tag{7}$$

where

$$\varphi_i(t,k)=c^+\delta f_i(t,k)+c^+\delta b_i(t,k)u_i(t,d). \tag{8}$$

In the algorithm the noise contains two parts: $I\{i\in Y_k\}\varphi_i(t,k)$ represents the structural error and $I\{i\in Y_k\}w_i(t+1,k)$ is the observation noise.

Theorem 2. *Assume* A1–A6 *hold for system* (1) *and criterion* (2), $i=1,\ldots,n$. *Then the control sequence* $\{u_i(t,k)\}$ *given by* (6) *is optimal.*

Proof. The proof is carried out by induction along the time axis t simultaneously for all subsystems.

Let i be fixed. For $t=0$ the algorithm (7) is written as

$$\delta u_i(0,k+1)=(1-\alpha_k h_k)\delta u_i(0,k)-\beta_k\epsilon_{k+1}, \tag{9}$$

where

$$\alpha_k\triangleq a(v(i,k)),\qquad \beta_k\triangleq a(v(i,k))I\{i\in Y_k\},$$
$$h_k\triangleq I\{i\in Y_k\}c^+b_i(0,k),\qquad \epsilon_{k+1}\triangleq\varphi_i(0,k)-w_i(1,k).$$

It is clear that $\sum_{k=1}^{\infty}\alpha_k=\infty$. By the definitions of $v(i,k)$ and Y_k, the nonzero elements of $\{\beta_k\}$ are in one-to-one correspondence with elements of $\{a(0),a(1),a(2),\ldots\}$. Therefore, we have $\sum_{k=1}^{\infty}\beta_k^2<\infty$. From

$$v(i,k)=v(i,l)+\sum_{j=l+1}^{k}I\{i\in Y_j\} \tag{10}$$

it follows that $v(i,k)\le v(i,j)+K$, and for $j=k-K+1,\ldots,k-1,k$ as $k\to\infty$

$$a(v(i,j))=a(v(i,k))\left(1-\frac{a(v(i,k))-a(v(i,j))}{a(v(i,k))}\right)$$
$$=a(v(i,k))(1+O(a(v(i,k)))),$$

i.e.,$\forall j=k-K+1,\ldots,k-1,k$

$$\alpha_j=\alpha_k(1+O(\alpha_k)). \tag{11}$$

Since $b_i(0,s)$ is continuous in s by A3, we have $b_i(0,k)\xrightarrow[k\to\infty]{}b_i(0,d)$ by A2 and $c^+b_i(0,k)$ converges to a positive constant by A4. Therefore, by A6 we have

$$\sum_{j=k-K+1}^{k}-h_j<-\gamma,\ \gamma>0 \tag{12}$$

for all sufficiently large k.

Set $\phi_{k,j}\triangleq(1-\alpha_k h_k)\cdots(1-\alpha_j h_j),\quad k\ge j,\ \phi_{j,j+1}\triangleq 1.$

It is clear that $(1-\alpha_j h_j)>0$ for all large enough j, say, $j\ge j_0$.

For any $k\ge j+K,\ j\ge j_0$ by (11) and (12) we have

$$\phi_{k,j}=\phi_{k-K,j}\left(1-\alpha_k\sum_{l=k-K+1}^{k}h_l+o(\alpha_k)\right)$$
$$\le\phi_{k-K,j}(1-\beta\alpha_k+o(\alpha_k))$$
$$=\phi_{k-K,j}\left(1-\frac{\beta}{K}\sum_{l=k-K+1}^{k}\alpha_l+o(\alpha_k)\right)$$
$$\le\exp\left(-c\sum_{l=k-K+1}^{k}\alpha_l\right)\phi_{k-K,j}\quad\text{with } c>0.$$

From here it follows that $\phi_{k,j}\le c_1\exp\left(-\frac{c}{2}\sum_{l=j}^{k}\alpha_l\right)\ \forall j\ge j_0$ for some $c_1>0$, and hence there is a c_2 such that

$$|\phi_{k,j}|\le c_2\exp\left(-\frac{c}{2}\sum_{l=j}^{k}\alpha_l\right)\quad\forall k\ge j+K,\ j\ge j_0.$$

Therefore, for $\forall k\ge j_0+K$, $\forall j\ge 0$ we have

$$|\phi_{k,j}|\le|\phi_{k,j_0}|\cdot|\phi_{j_0-1,j}|\le c_0\exp\left(-\frac{c}{2}\sum_{l=j}^{k}\alpha_l\right) \tag{13}$$

for some $c_0>0$.

From (9) it follows that

$$\delta u_i(0,k+1)=\phi_{k,0}\delta u_i(0,0)+\sum_{j=0}^{k}\phi_{k,j+1}\beta_j\epsilon_{j+1}, \tag{14}$$

where the first term at the right-hand of (14) tends to zero as $k\to\infty$ because of (13). By A2 and A3 it is seen that $\varphi_i(0,k)\xrightarrow[k\to\infty]{}0$. By A5 it follows that

$$\sum_{k=1}^{\infty}\beta_k w_i(1,k)<\infty. \tag{15}$$

Since $\epsilon_{k+1}\triangleq\varphi_i(0,k)-w_i(1,k)$, the last term of (14) also tends to zero as $k\to\infty$. For this, the proof is given in Lemma 3.1.1 of Chen (2002). Thus, we have proved the optimality of the control sequence $u_i(0,k)\xrightarrow[k\to\infty]{}u_i(0,d)$.

Inductively, assume the optimality takes place for $t=0,1,\ldots,s-1$. We now show it is also true for $t=s$.

By the inductive assumption we have $\delta u_i(t,k) \xrightarrow[k\to\infty]{} 0$, $i = 1,\ldots,n$, $t = 0,1,\ldots,s-1$, and by Lemma 1

$$\delta x_i(s,k) \xrightarrow[k\to\infty]{} 0, \qquad \delta f_i(s,k) \xrightarrow[k\to\infty]{} 0,$$
$$\delta b_i(s,k) \xrightarrow[k\to\infty]{} 0, \quad \forall i = 1,2,\ldots,n. \tag{16}$$

Then, it follows that $\varphi_i(s,k) \xrightarrow[k\to\infty]{} 0$, $\forall i = 1,2,\ldots,n$. As for the case $t = 0$, a similar treatment leads to $u_i(s,k) \xrightarrow[k\to\infty]{} u_i(s,d)$, $\forall i$. This proves optimality of control for $t = s$ and completes the proof of the theorem. □

We now consider systems with communication delays. For many practical systems the information at the kth iteration of ith subsystem may be forwarded to the jth subsystem not at the $(k+1)$th iteration but at a later iteration.

Let the ith subsystem of a large scale system be described as follows:

$$\begin{cases} x_i(t+1,k) = f_i(t,\bar{x}(t,k)) + b_i(t,\bar{x}(t,k))u_i(t,k), \\ y_i(t,k) = c_i(t)x_i(t,k) + w_i(t,k), \end{cases} \tag{17}$$

where

$$\bar{x}(t,k) \triangleq [x_1^T(t,k-\tau_{1i}(k)),\ldots,x_n^T(t,k-\tau_{ni}(k))]^T, \tag{18}$$

and where $\tau_{ji}(k) > 0$, $j \neq i$ denotes the random delay for the ith subsystem at iteration k to receive information from the jth subsystem, while each subsystem receives information from itself without any delay, i.e., $\tau_{ii} = 0$. In other words, at the kth iteration the latest information from the jth subsystem obtained by the ith subsystem is $x_j(t,k-\tau_{ji}(k))$, and no information from $x_j(t,m)$ with $m > k-\tau_{ji}(k)$ can reach the ith subsystem.

For the delay $\tau_{ji}(k)$ the following condition is used.

A7. There is an integer M such that $\tau_{ji}(k) < M$ $\forall j,i,k$.

Theorem 3. *Assume* A1–A7 *hold for system* (17) *and criterion* (2), *$i = 1,\ldots,n$. Then, the control $\{u_i(t,k)\}$ given by* (6) *is optimal.*

Sketch of the proof. It is intuitively understandable that convergence will remain true if delays exist in some subsystems whenever the delays are uniformly bounded. This is because the information will arrive sooner or later. More precisely, in the ith subsystem the influence caused by delays $\{\tau_{ji}(k)\}$ is contained in $\varphi_i(t,k)$ given by (8), and in the convergence analysis of ILC the key step is to show $\varphi_i(t,k) \xrightarrow[k\to\infty]{} 0$. Paying attention to (17) and (18), we see that $\varphi_i(t,k)$ tends to zero as $k \to \infty$ no matter $\tau_{ji}(k) = 0$ or $\tau_{ji}(k) > 0$ but $\{\tau_{ji}(k)\}$ is bounded. Therefore, the proof of the theorem is completely similar to that for Theorem 2. □

Remark 2. In A4 we assume that the sign of $c^+b_i(t,k)$ is known, but we may consider the more general case where the sign of $c^+b_i(t,k)$ is unknown. For this we have to modify the algorithm (6) by applying the expanding truncation technique proposed in Chen (2002). The idea is as follows: if the control is switched in the wrong direction, then the system output will diverge and hit the truncation bound. Then the control switches to the opposite direction and simultaneously enlarge the truncation bound. It is proved that the truncation ceases in a finite steps, and the algorithm becomes (6) with correct control direction. This again yields optimality of control.

5. Illustrative example

For a large scale system the number of subsystems n is a large number, but here for convenience of illustration let us take $n = 3$. The three subsystems, indexed by subscripts 1, 2, and 3, are described as follows:

$$x_1(t+1,k) = 1.05x_1(t,k) + 0.2\sin(x_2(t,k-\tau_{21}(k))) + 0.45u_1(t,k),$$
$$y_1(t,k) = x_1(t,k) + w_1(t,k),$$
$$x_2(t+1,k) = 1.1x_2(t,k) + 0.2\sin(x_3(t,k-\tau_{32}(k))) + 0.5u_2(t,k),$$
$$y_2(t,k) = x_2(t,k) + w_2(t,k),$$
$$x_3(t+1,k) = 0.33\sin(x_1(t,k-\tau_{13}(k))) + 1.15x_3(t,k) + 0.8u_3(t,k),$$
$$y_3(t,k) = x_3(t,k) + w_3(t,k),$$

where time t is valued in $t \in \{1,\ldots,8\}$. Assume $w_i(t,k) \in \mathcal{N}(0, 0.01)$, $i = 1,2,3$. The random communication delay $\tau_{ij}(k)$ from the ith subsystem to the jth subsystem, $i,j \in \{1,2,3\}$ takes values 1 and 0 with probability $p = 0.5$, respectively.

To simulate data missing, i.e., to realize random asynchronous control updating with A6 satisfied, let us fix $K = 3$ in A6. Iteration steps are separated into groups of three successive steps, i.e., $\{1,2,3\}$, $\{4,5,6\}$, $\{7,8,9\}$, …, and randomly select one

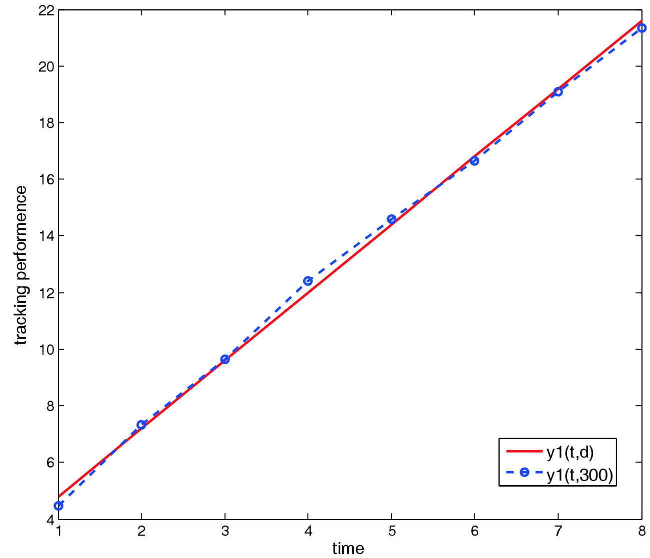

Fig. 1. $y_1(t,300)$ vs. $y_1(t,d)$.

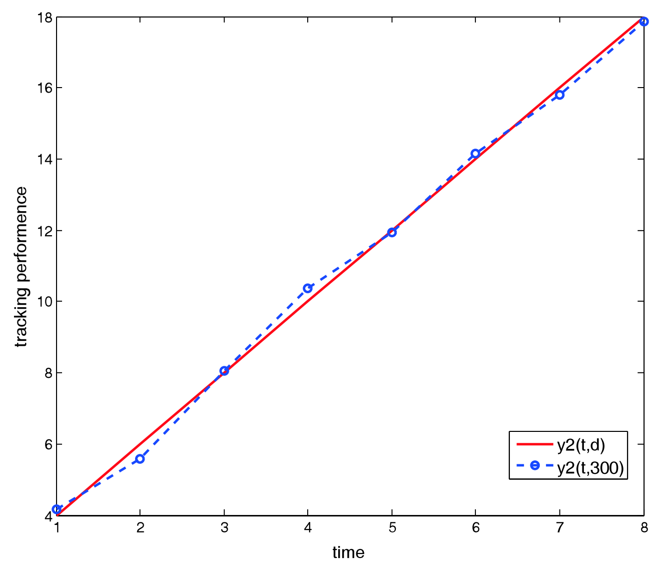

Fig. 2. $y_2(t,300)$ vs. $y_2(t,d)$.

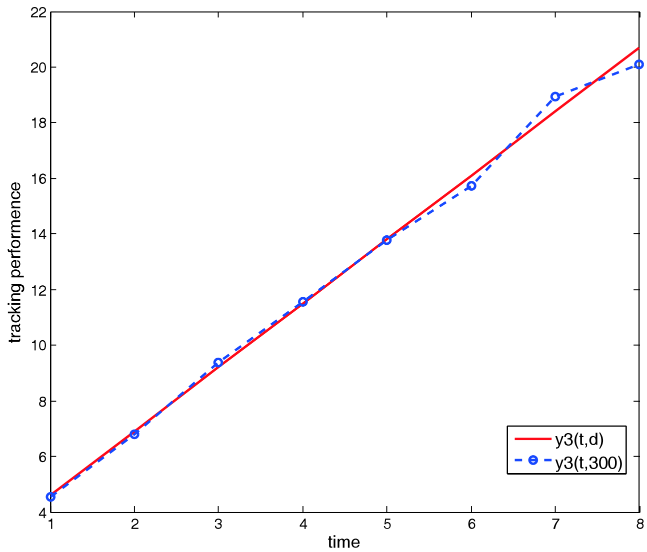

Fig. 3. $y_3(t, 300)$ vs. $y_3(t, d)$.

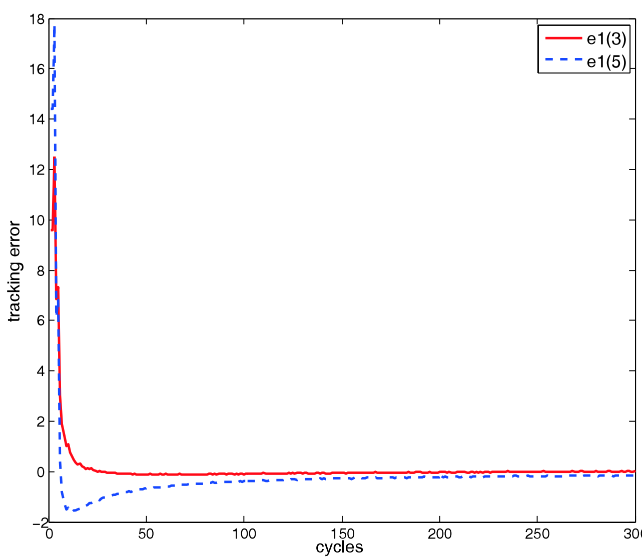

Fig. 4. Tracking errors $e_1(t, k)$ for $t = 3$ and $t = 5$.

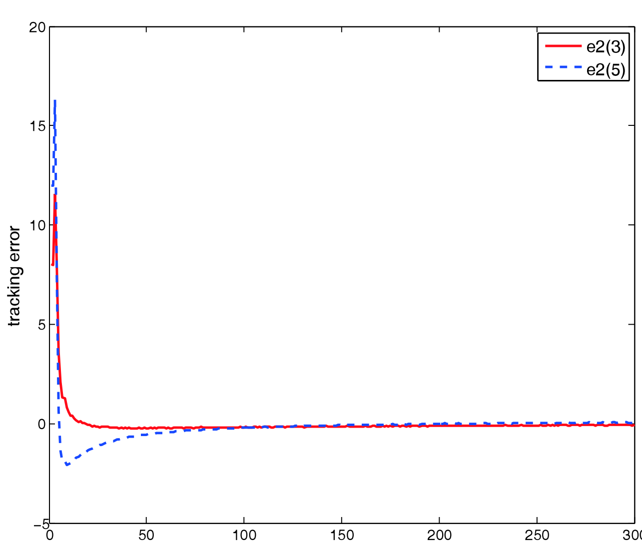

Fig. 5. Tracking errors $e_2(t, k)$ for $t = 3$ and $t = 5$.

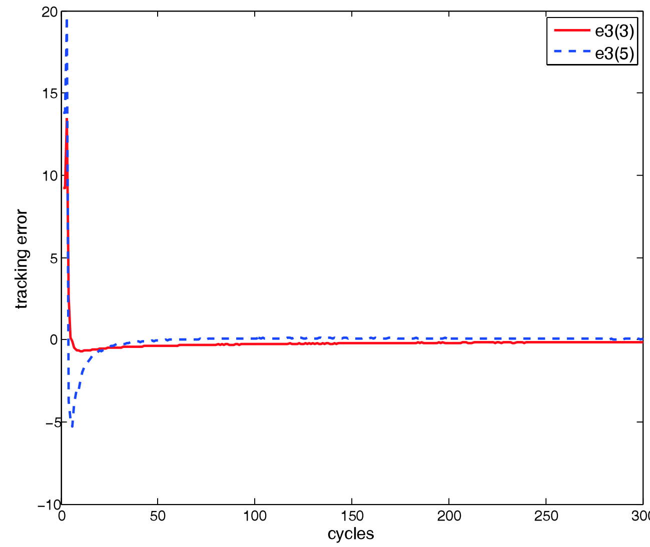

Fig. 6. Tracking errors $e_3(t, k)$ for $t = 3$ and $t = 5$.

step from each group, say, 1, 5, 9, ... For these elected steps the control is not updated.

The reference signals are $y_1(t, d) = 2.4t$, $y_2(t, d) = 2t$, and $y_3(t, d) = 2.3t$, $t \in \{1, \dots, 8\}$ for subsystems 1, 2, and 3, respectively. The initial control actions are $u_1(t, 0) = u_2(t, 0) = u_3(t, 0) \triangleq 0$, $\forall t \in \{1, \dots, 8\}$.

The algorithm has run 300 cycles and the tracking results at $k = 300$ are presented in Figs. 1–3 for subsystems 1, 2, and 3, respectively, where the solid lines are the reference signals and the cycle-dashed lines denote the system outputs.

The tracking errors at $t = 3$ and $t = 5$ are demonstrated in Figs. 4–6 for subsystems 1, 2, and 3, respectively.

From the figures we see that the ILC designed in the paper works well.

6. Concluding remarks

The ILC is considered for the large scale systems with possible data missing in transmission and with asynchronous control updating. Each subsystem updates its control, based on its own input–output information and also on the tracking target. The convergence with probability one of control to the optimal one is established. Similar results are also derived for systems with delays and unknown control direction. The simulation results are consistent with theoretical analysis.

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