International Journal of Systems Science, 2016
Vol. 47, No. 10, 2290–2301, http://dx.doi.org/10.1080/00207721.2014.992494

Reliable H_∞ control of discrete-time systems against random intermittent faults

Yuan Tao, Dong Shen, Mengqi Fang and Youqing Wang*

College of Information Science and Technology, Beijing University of Chemical Technology, Beijing, P.R. China

(*Received 6 June 2014; accepted 24 November 2014*)

A passive fault-tolerant control strategy is proposed for systems subject to a novel kind of intermittent fault, which is described by a Bernoulli distributed random variable. Three cases of fault location are considered, namely, sensor fault, actuator fault, and both sensor and actuator faults. The dynamic feedback controllers are designed not only to stabilise the fault-free system, but also to guarantee an acceptable performance of the faulty system. The robust H_∞ performance index is used to evaluate the effectiveness of the proposed control scheme. In terms of linear matrix inequality, the sufficient conditions of the existence of controllers are given. An illustrative example indicates the effectiveness of the proposed fault-tolerant control method.

Keywords: fault-tolerant control; random intermittent fault; robust H_∞ performance; sensor fault; actuator fault

1. Introduction

Over the past three decades, to satisfy the growing demands on safety and reliability, the fault-tolerant control system has attracted considerable attention (Zhang & Jiang, 2008), especially in aircraft systems, industrial processes, and other safety-critical systems (Chilin, Liu, Davis, & Christofides, 2012; Gao, Jiang, Qi, & Xu, 2011; Hua, Ding, & Guan, 2012; Hu & Xiao, 2013). From the viewpoint of the control design, the fault-tolerant control design can be classified either as an active method (Dong, Zhong, & Ding, 2012; Guenab, Weber, Theilliol, & Zhang, 2011; Yao, Qin, Wang, & Jiang, 2012) or as a passive one (Tabatabaeipour, Izadi-Zamanabadi, Bak, & Ravn, 2012; Tian, Yue, & Peng, 2010; Wang & Ju, 2013). To achieve the purpose of active fault-tolerant control, fault detection and diagnosis, which itself also draws considerable attention in the control community (He, Wang, Liu, & Zhou, 2013; Hwang, Kim, Kim, & Seah, 2010), is necessary to provide real-time fault information to reconfigure the controller. In contrast, the design of passive fault-tolerant control relies on a-priori knowledge of malfunction information (e.g. location, amplitude).

Hitherto research on fault-tolerant control has covered various kinds of plants, from linear systems (Mahmoud, Jiang, & Zhang, 2001) to nonlinear ones (Gassara, El Hajjaji, Kchaou, & Chaabane, 2014; Wang, Zhou, & Gao, 2007), from continuous-time systems (Seron, De Dona, & Olaru, 2012) to discrete-time ones (Liu, Wang, & Wang, 2011; Witczak, Puig, & Montes de Oca, 2013), as well as from continuous processes (Kargar, Salahshoor, & Yazdanpanah, 2014) to batch ones (Wang, Shi, Zhou, & Gao, 2006). Additionally, both sensor faults and actuator faults have been considered in a number of previous studies. However, most faults considered are permanent faults. In other words, once the faults occur, they would not disappear until manual intervention. As a matter of fact, in many practical systems such as electronic systems, aircraft systems, mechanic systems, and communication systems, there exist lots of intermittent faults.

Even though there is no unified and well-accepted definition of an intermittent fault at present, as opposed to a permanent fault, it can be described as a fault of intermittence and recurrence (Correcher, Garcia, Morant, Quiles, & Rodriguez, 2012; Zhou, Shi, & He, 2014). Since the feature of an intermittent fault is different from that of a permanent fault, the traditional fault detection and diagnosis method usually does not prove effective; thus, the active fault-tolerant control of an intermittent fault is hard to realise. As far as the passive fault-tolerant control is concerned, it does not need online fault detection and diagnosis, which benefits in dealing with an intermittent fault. To authors' best knowledge, there are few reported studies on fault-tolerant control of an intermittent fault (Gu, Zhang, & Du, 2011; Xu, Yang, Jiang, Zhou, & Zhang, 2014; Yang, Jiang, & Zhang, 2012), and this motivates this paper.

This paper investigates the passive fault-tolerant control problem for discrete-time systems subject to a random intermittent fault. It aims to design reliable controllers to ensure the stability of fault-free closed-loop systems and the desired performance of a faulted system. The intermittent fault discussed in this paper is formulated as a probabilistic event and is additive to the normal signal of a sensor or actuator in systems. The occurrence of the intermittent fault

*Corresponding author. Email: wang.youqing@ieee.org

is assumed to satisfy the Bernoulli random binary distribution which takes values of either 0 or 1. By using the linear matrix inequality (LMI) approach, full-dimension dynamic feedback controllers are obtained such that, for considered random intermittent faults, the closed-loop systems are stable and satisfy the corresponding H_∞ performance required.

The rest of the paper is organised as follows. In Section 2, the problem formulation and preliminaries are given. Then, the main results of reliable control for discrete-time systems against random intermittent faults are presented in Section 3, together with elaborate proofs. Section 4 provides a numerical example to verify the effectiveness of the proposed controllers. Some conclusions are drawn in Section 5 which closes the paper.

Notations: Throughout the paper, for real symmetric matrices, the notation $X > Y$ means the matrix $X - Y$ is positive definite. $\mathbf{R}^n$ represents the n-dimension Euclidean space while $\mathbf{R}^{n\times m}$ is the set of all $n \times m$ real matrices. The superscript T denotes the transpose. $\mathbb{E}\{x\}$ stands for the expectation of x. Prob$\{\circ\}$ represents the occurrence probability of the event '$\circ$'. diag$\{M_1, M_2, \ldots, M_n\}$ denotes a block diagonal matrix which is composed of square matrices $M_1, M_2, \ldots, M_n$. $l_2[0, \infty)$ is the space of square summable vectors. The symbol '$*$' represents the corresponding transposed block in the symmetry block matrix.

2. Problem formulation and preliminaries

Consider the following system:

$$\begin{cases} x(k+1) = Ax(k) + B(u(k) + \delta_a(k) f_a(k)), \\ y(k) = C_1 x(k) + \delta_s(k) f_s(k), \\ z(k) = C_2 x(k), \end{cases} \tag{1}$$

where $x(k) \in \mathbf{R}^n$, $u(k) \in \mathbf{R}^m$, and $y(k) \in \mathbf{R}^p$ represent the state, input, and measured output, respectively, and $z(k) \in \mathbf{R}^r$ is the controlled output. A, B, C_1, and C_2 are known matrices of appropriate dimensions.

Faults could occur both at the sensor side and the actuator side, which are denoted as $f_s \in \mathbf{R}^p$ and $f_a \in \mathbf{R}^m$, respectively. The occurrences of the two faults are determined by two random variables, both of which obey the Bernoulli distribution and take values on 0 or 1. Define δ_s for a sensor fault and δ_a for an actuator fault, satisfying the following requirements:

$$\begin{cases} \Pr\text{ob}\{\delta_s(k) = 1\} = \alpha, \\ \Pr\text{ob}\{\delta_s(k) = 0\} = 1 - \alpha, \end{cases} \tag{2}$$

$$\begin{cases} \Pr\text{ob}\{\delta_a(k) = 1\} = \beta, \\ \Pr\text{ob}\{\delta_a(k) = 0\} = 1 - \beta, \end{cases} \tag{3}$$

where α and β are known parameters. In this study, three cases of fault occurrences are discussed in sequence: only sensor fault, only actuator fault, and both of them.

Remark 1: Intermittent faults widely exist in many practical systems, such as electronic systems, aircraft systems, mechanic systems, and communication systems. Many of these faults are in the nature of randomness. The majority of the random intermittent faults may result from the system itself, such as net congestion in networked systems and electromagnetic interference in electronic systems, and some of them are caused by the environment. Because it is difficult to describe these faults in a deterministic fashion, stochastic variables are widely employed to depict them (He, Hu, & Peng, 2013; Xu et al., 2014; Yang et al., 2012).

Remark 2: The description of the random intermittent fault is similar to that of a missing measurement in a network control system (He, Wang, Ji, & Zhou, 2011; Wang, Shen, Shu, & Wei, 2012). However, there exist evident differences between these two descriptions. Obviously, a missing measurement in He et al. (2011) and Wang et al. (2012) can be regarded as a multiplicative fault. In contrast, the fault described in this paper is a typical additive one. In addition, the fault scenarios of the proposed description could contain the missing measurement with a proper f_s in (1). Theoretically, with the proposed random intermittent fault description, the fault itself can be of any form, such as a square wave, a triangular wave, or any other regular or irregular signals. However, in the framework of LMI, we assume that the considered fault is square summable, i.e. f belongs to $l_2[0, \infty)$.

Consider the following full-order dynamic feedback controller:

$$\begin{cases} x_c(k+1) = A_c x_c(k) + B_c y(k), \\ u(k) = C_c x_c(k), \end{cases} \tag{4}$$

where A_c, B_c, and C_c are parameters that need to be designed.

Define

$$\eta(k) = \begin{bmatrix} x(k) \\ x_c(k) \end{bmatrix}. \tag{5}$$

Three closed-loop systems can be constructed by substituting (4) and (5) into a corresponding faulty scenario description revised from (1). A general form of the closed-loop system is summarised as

$$\begin{cases} \eta(k+1) = \bar{A}_g \eta(k) + \bar{B}_g \delta_g(k) f_g(k), \\ z(k) = \bar{C}_g \eta(k), \end{cases} \tag{6}$$

where the subscript 'g' represents the general form; $\bar{A}_g$, $\bar{B}_g$, $\bar{C}_g$ are the parameters of the augmented closed-loop system; and $f_g(k)$ and $\delta_g(k)$ stand for the considered fault and relevant random variables.

In the framework of reliable H_∞ control, the aim of this work is to design a dynamic output feedback controller for the considered faulty system, such that the closed-loop system (6) satisfies the following requirements:

(1) In the condition of $\delta_g(k) = 0$, the closed-loop system (6) is asymptotically stable.
(2) Under the zero-initial condition, the controller output satisfies

$$\sum_{k=0}^{\infty} \mathbb{E}\{|z(k)|^2\} < \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\{\|\delta_g(k) f_g(k)\|^2\},$$

where $\gamma > 0$ is a prescribed scalar.

3. Main results

In this section, the sufficient conditions are first presented and proved for a passive fault-tolerant controller for systems subject to a random intermittent sensor fault. Subsequently, the case for a random intermittent actuator fault is studied. Finally, both these kinds of random intermittent faults are considered simultaneously.

3.1. Case A: sensor fault

Consider the following system description which involves only a random intermittent sensor fault:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k), \\ y(k) = C_1 x(k) + \delta_s(k) f_s(k), \\ z(k) = C_2 x(k), \end{cases} \tag{7}$$

where all the parameters have the same definition as those defined in Equation (1) in the previous section.

Sorting out (4) and (7), and using the definition (5), an augmented closed-loop system can be derived as

$$\begin{cases} \eta(k+1) = \bar{A}\eta(k) + \delta_s(k)\bar{B}_2 f_s(k) \\ \quad = \bar{A}\eta(k) + \bar{B}_1 f_s(k) + (\delta_s(k) - \alpha)\bar{B}_2 f_s(k), \\ z(k) = \bar{C}\eta(k), \end{cases} \tag{8}$$

where

$$\bar{A} = \begin{bmatrix} A & BC_c \\ B_c C_1 & A_c \end{bmatrix}, \bar{B}_1 = \begin{bmatrix} \mathbf{0} \\ \alpha B_c \end{bmatrix}, \bar{B}_2 = \begin{bmatrix} \mathbf{0} \\ B_c \end{bmatrix},$$
$$\bar{C} = [C_2 \ \mathbf{0}]. \tag{9}$$

The control objective is to design a dynamic controller of the form (4) such that

(1) in the condition of $\delta_s(k) = 0$, the closed-loop system (8) is asymptotically stable; and
(2) under the zero-initial condition, the controlled output satisfies

$$\sum_{k=0}^{\infty} \mathbb{E}\left\{\|z(k)\|^2\right\} < \gamma_s^2 \sum_{k=0}^{\infty} \mathbb{E}\left\{\|\delta_s(k) f_s(k)\|^2\right\}. \tag{10}$$

Theorem 1: *For a prescribed scalar $\gamma_s > 0$ and controller parameters A_c, B_c, and C_c, if there exists a positive-definite matrix $P = P^T$ such that the following matrix inequality constraint holds:*

$$\begin{bmatrix} -P & \mathbf{0} & P\bar{A} & P\bar{B}_1 & \mathbf{0} & \mathbf{0} \\ * & -I & \tilde{C} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -P & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\gamma_s^2 \alpha I & \mathbf{0} & \mu_1 \bar{B}_2^T P \\ * & * & * & * & -I & \mathbf{0} \\ * & * & * & * & * & -P \end{bmatrix} < 0, \tag{11}$$

where $\mu_1 = [(1-\alpha)\alpha]^{1/2}$, then the control objective of the closed-loop system is satisfied.

Proof: Applying the Schur complement lemma to (11) leads to

$$\begin{bmatrix} \bar{A}^T P\bar{A} - P + \bar{C}^T\bar{C} & \bar{A}^T P\bar{B}_1 \\ \bar{B}_1^T P\bar{A} & \bar{B}_1^T P\bar{B}_1 + \mu_1^2 \bar{B}_2^T P\bar{B}_2 - \gamma_s^2 \alpha I \end{bmatrix} < 0. \tag{12}$$

This further implies

$$\bar{A}^T P\bar{A} - P < 0. \tag{13}$$

Define the following Lyapunov function:

$$V(k) = \eta^T(k) P\eta(k). \tag{14}$$

If $\delta_s(k) = 0$, the difference of (14) is

$$\begin{aligned} \Delta V(k) &:= V(\eta(k+1)) - V(\eta(k)) \\ &= \left(\bar{A}\eta(k)\right)^T P\left(\bar{A}\eta(k)\right) - \eta^T(k) P\eta(k) \\ &= \eta^T(k)\bar{A}^T P\bar{A}\eta(k) - \eta^T(k) P\eta(k) \\ &= \eta^T(k)\left(\bar{A}^T P\bar{A} - P\right)\eta(k). \end{aligned} \tag{15}$$

Noticing (13), the value of $\Delta V(k)$ is strictly negative. Thus, one can conclude that the closed-loop system (8) is asymptotically stable.

Moreover, it can be derived from (12) that

$$\mathbb{E}\{V(\eta(k+1))\} - \mathbb{E}\{V(\eta(k))\} + \mathbb{E}\left\{z^T(k) z(k)\right\}$$
$$- \gamma_s^2 \mathbb{E}\left\{\delta_s^2(k) f_s^T(k) f_s(k)\right\}$$

$$
\begin{aligned}
&= \eta^T(k)\bar{A}^T P\bar{A}\eta(k) + \eta^T(k)\bar{A}^T P\bar{B}_1 f_s(k) \\
&\quad + f_s^T(k)\bar{B}_1^T P\bar{A}\eta(k) + f_s^T(k)\bar{B}_1^T P\bar{B}_1 f_s(k) \\
&\quad + \mathbb{E}\left\{(\delta_s(k)-\alpha)^2\right\} f_s^T(k)\bar{B}_2^T P\bar{B}_2 f_s(k) \\
&\quad - \eta^T(k)P\eta(k) + \eta^T(k)\bar{C}^T\bar{C}\eta(k) - \gamma_s^2\alpha f_s^T(k)f_s(k) \\
&= \begin{bmatrix} \eta(k) \\ f_s(k) \end{bmatrix}^T \\
&\quad \times \begin{bmatrix} \bar{A}^T P\bar{A} - P + \bar{C}^T\bar{C} & \bar{A}^T P\bar{B}_1 \\ \bar{B}_1^T P\bar{A} & \bar{B}_1^T P\bar{B}_1 + \mu_1^2\bar{B}_2^T P\bar{B}_2 - \gamma_s^2\alpha I \end{bmatrix} \\
&\quad \times \begin{bmatrix} \eta(k) \\ f_s(k) \end{bmatrix} < 0.
\end{aligned}
$$

It means that

$$
\begin{aligned}
&\mathbb{E}\{V(\eta(k+1))\} - \mathbb{E}\{V(\eta(k))\} + \mathbb{E}\left\{z^T(k)z(k)\right\} \\
&\quad - \gamma_s^2\mathbb{E}\left\{\delta_s^2(k)f_s^T(k)f_s(k)\right\} < 0. \qquad (16)
\end{aligned}
$$

By summing inequality (16) from $k = 0$ to ∞, one has

$$
\begin{aligned}
&\sum_{k=0}^{\infty}\mathbb{E}\left\{\|z(k)\|^2\right\} < \gamma_s^2\sum_{k=0}^{\infty}\mathbb{E}\left\{\|\delta_s(k)f_s(k)\|^2\right\} \\
&\quad + \mathbb{E}\{V(0)\} - \mathbb{E}\{V(\infty)\}. \qquad (17)
\end{aligned}
$$

Hence, for the closed-loop system (8) with the zero-initial condition, the following inequality constraint can be satisfied:

$$
\sum_{k=0}^{\infty}\mathbb{E}\left\{\|z(k)\|^2\right\} < \gamma_s^2\sum_{k=0}^{\infty}\mathbb{E}\left\{\|\delta_s(k)f_s(k)\|^2\right\}. \qquad (18)
$$

This completes the proof. □

It is evident that the inequality constraint (11) is a bilinear matrix inequality (BMI) rather than a standard LMI. To solve the BMI, the following theorem is proposed, and then the controller parameters can be obtained by solving an LMI (Scherer, Gahinet, & Chilali, 1997).

Theorem 2: *For a prescribed scalar $\gamma_s > 0$, if there exist positive-definite matrices $X = X^T$ and $Y = Y^T$ and real matrices $\hat{A}$, $\hat{B}$, and $\hat{C}$ such that the following LMI holds:*

$$
\begin{bmatrix}
-X & -I & \mathbf{0} & AX + B\hat{C} & A & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
* & -Y & \mathbf{0} & \hat{A} & YA + \hat{B}C_1 & \alpha\hat{B} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
* & * & -I & C_2X & C_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
* & * & * & -X & -I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
* & * & * & * & -Y & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
* & * & * & * & * & -\gamma_s^2\alpha I & \mathbf{0} & \mathbf{0} & \mu_1\hat{B}^T \\
* & * & * & * & * & * & -I & \mathbf{0} & \mathbf{0} \\
* & * & * & * & * & * & * & -X & -I \\
* & * & * & * & * & * & * & * & -Y
\end{bmatrix} < 0, \qquad (19)
$$

where $\mu_1 = [(1-\alpha)\alpha]^{1/2}$, and $\hat{A}$, $\hat{B}$, and $\hat{C}$ are auxiliary variables, which can be used to calculate the controller, then the controller parameters are designed as

$$
\begin{cases}
A_c = N^{-1}(\hat{A} - \hat{B}C_1X - YB\hat{C} - YAX)M^{-T}, \\
B_c = N^{-1}\hat{B}, \\
C_c = \hat{C}M^{-T},
\end{cases} \qquad (20)
$$

where

$$
MN^T = I - XY. \qquad (21)
$$

Then, the control objective of the closed-loop system is satisfied.

Proof: P and P^{-1} can be partitioned as

$$
P = \begin{bmatrix} Y & N \\ N^T & \star \end{bmatrix}, \; P^{-1} = \begin{bmatrix} X & M \\ M^T & \star \end{bmatrix}, \qquad (22)
$$

where X and Y are symmetric with a dimension of are symmetric with a dimension of $n \times n$.

Construct matrices Π_1 and Π_2 with the following form:

$$
\Pi_1 = \begin{bmatrix} X & I \\ M^T & \mathbf{0} \end{bmatrix}, \; \Pi_2 = \begin{bmatrix} I & Y \\ \mathbf{0} & N^T \end{bmatrix}. \qquad (23)
$$

Based on the fact that $PP^{-1} = I$, (22) and (23) imply that

$$
P\Pi_1 = \Pi_2. \qquad (24)
$$

Let

$$
\begin{cases}
\hat{A} = NA_cM^T + NB_cC_1X + YBC_cM^T + YAX, \\
\hat{B} = NB_c, \\
\hat{C} = C_cM^T.
\end{cases} \qquad (25)
$$

According to (9), (24), and (25), the following equalities can be calculated:

$$\begin{cases} \prod_1^T P \prod_1 = \prod_1^T \prod_2 = \begin{bmatrix} X & I \\ I & Y \end{bmatrix}, \\ \prod_1^T P \bar{A} \prod_1 = \prod_2^T \bar{A} \prod_1 = \begin{bmatrix} AX + B\hat{C} & A \\ \hat{A} & YA + \hat{B}C_1 \end{bmatrix}, \\ \prod_1^T P \bar{B} = \prod_2^T \bar{B} = \begin{bmatrix} \mathbf{0} \\ \hat{B} \end{bmatrix}, \\ \bar{C} \prod_1 = \begin{bmatrix} C_2 X & C_2 \end{bmatrix}. \end{cases} \tag{26}$$

Then, performing the congruence transformation by diag$\left(\prod_1, I, \prod_1, I, I, \prod_1\right)$ to (11) leads to the LMI (19) with $\hat{A}$, $\hat{B}$, $\hat{C}$, X, and Y to be solved. Hence, if (19) is satisfied, the condition (11) holds.

If LMI (19) is feasible, one gets

$$\begin{pmatrix} X & I \\ I & Y \end{pmatrix} > 0. \tag{27}$$

According to (27) and the Schur complement lemma, it is easy to get $Y > 0$ and $X - Y^{-1} > 0$ which make $I - XY$ nonsingular. Recalling (24), it follows directly that $MN^T = I - XY$. Then, M and N always can be calculated by applying singular value decomposition (SVD) to $I - XY$. Consequently, controller parameters (20) are obtained from (25), which completes the proof. □

3.2. Case B: actuator fault

Consider the following system which contains only a random intermittent actuator fault:

$$\begin{cases} x(k+1) = Ax(k) + B(u(k) + \delta_a(k) f_a(k)), \\ y(k) = C_1 x(k), \\ z(k) = C_2 x(k). \end{cases} \tag{28}$$

By using the definition (5) and sorting out (28) and (4), the following closed-loop system can be derived:

$$\begin{cases} \eta(k+1) = \bar{A}\eta(k) + \delta_a(k)\bar{B}_1 f_a(k) \\ \qquad\qquad = \bar{A}\eta(k) + \bar{B}_1 f_a(k) + (\delta_a(k) - \beta)\bar{B}_2 f_a(k), \\ z(k) = \bar{C}\eta(k), \end{cases} \tag{29}$$

where

$$\bar{A} = \begin{bmatrix} A & BC_c \\ B_c C_1 & A_c \end{bmatrix}, \bar{B}_1 = \begin{bmatrix} \beta B \\ \mathbf{0} \end{bmatrix}, \bar{B}_2 = \begin{bmatrix} B \\ \mathbf{0} \end{bmatrix},$$
$$\bar{C} = [C_2 \ \mathbf{0}]. \tag{30}$$

Then, the control objective is to design a dynamic controller of the form (4) such that

(1) in the condition of $\delta_a(k) = 0$, the closed-loop system (29) is asymptotically stable; and
(2) under the zero-initial condition, the controlled output $z(k)$ satisfies

$$\sum_{k=0}^{\infty} \mathbb{E}\left\{\|z(k)\|^2\right\} < \gamma_a^2 \sum_{k=0}^{\infty} \mathbb{E}\left\{\|\delta_a(k) f_a(k)\|^2\right\}. \tag{31}$$

Theorem 3: *For a prescribed scalar $\gamma_a > 0$ and controller parameters A_c, B_c, and C_c, if there exists a positive-definite matrix $P = P^T$ such that the following matrix inequality constraint holds:*

$$\begin{bmatrix} -P & \mathbf{0} & P\bar{A} & P\bar{B}_1 & \mathbf{0} & \mathbf{0} \\ * & -I & \bar{C} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -P & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\gamma_a^2\beta I & \mathbf{0} & \mu_2\bar{B}_2^T P \\ * & * & * & * & -I & \mathbf{0} \\ * & * & * & * & * & -P \end{bmatrix} < 0, \tag{32}$$

where $\mu_2 = [(1-\beta)\beta]^{1/2}$, then the control objective is satisfied.

Proof: From inequality (32), one gets the following inequality (33) by the Schur complement lemma:

$$\begin{bmatrix} \bar{A}^T P\bar{A} - P + \bar{C}^T\bar{C} & \bar{A}^T P\bar{B}_1 \\ \bar{B}_1^T P\bar{A} & \bar{B}_1^T P\bar{B}_1 + \mu_2^2\bar{B}_2^T P\bar{B}_2 - \gamma_a^2\beta I \end{bmatrix} < 0. \tag{33}$$

This further implies $\bar{A}^T P\bar{A} - P < 0$. Therefore, by choosing a Lyapunov function the same as (14), it is obvious that the difference of the Lyapunov function $\Delta V(k)$ is negative under the condition of $\delta_a(k) = 0$. Then, the closed-loop system (29) is asymptotically stable when $\delta_a(k) = 0$.

Similar to the steps in the proof of Theorem 1, the following inequality is derived based on (33):

$$\mathbb{E}\{V(\eta(k+1))\} - \mathbb{E}\{V(\eta(k))\} + \mathbb{E}\left\{z^T(k)z(k)\right\} - \gamma_a^2\mathbb{E}\left\{\delta_a^2(k) f_a^T(k) f_a(k)\right\} < 0. \tag{34}$$

By summing inequality (34) from $k = 0$ to ∞, the control performance index (31) is inferred under the zero-initial condition which completes the proof. □

Theorem 4: *For a prescribed scalar $\gamma_a > 0$, if there exist positive-definite matrices $X = X^T$, $Y = Y^T$, and real matrices $\hat{A}$, $\hat{B}$, and $\hat{C}$ such that the following LMI holds:*

$$\begin{bmatrix} -X & -I & \mathbf{0} & AX+B\hat{C} & A & \beta B & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & -Y & \mathbf{0} & \hat{A} & YA+\hat{B}C_1 & \beta YB & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -I & C_2X & C_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & -X & -I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & -Y & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & -\gamma_a^2\beta I & \mathbf{0} & \mu_2 B^T & \mu_2 B^T Y \\ * & * & * & * & * & * & -I & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & * & * & -X & -I \\ * & * & * & * & * & * & * & * & -Y \end{bmatrix} < 0, \quad (35)$$

where $\mu_2 = [(1-\beta)\beta]^{1/2}$, then the closed-loop system (29) is asymptotically stable in the condition of $\delta_a(k) = 0$, and inequality (31) can be satisfied under zero-initial condition. Moreover, the controller parameters can be determined by (20) and (21).

Proof: With the variables P, P^{-1}, Π_1, Π_2, $\hat{A}$, $\hat{B}$, and $\hat{C}$ defined in (22)–(25) and the equivalent connections described by (26), conducting the congruence transformation by diag $\left(\prod_1, I, \prod_1, I, I, \prod_1\right)$ to (32) leads to the LMI (35). Similar to the proof of Theorem 2, controller parameters (20) are obtained from (25). This completes the proof. □

3.3. Case C: both sensor and actuator faults

Consider the following system which encompasses both a random intermittent sensor fault and an actuator fault:

$$\begin{cases} x(k+1) = Ax(k) + B(u(k) + \delta_a(k) f_a(k)), \\ y(k) = C_1 x(k) + \delta_s(k) f_s(k), \\ z(k) = C_2 x(k). \end{cases} \quad (36)$$

Using the definition (5) and rearranging the system description (36) and controller (4), the closed-loop system can be rewritten as

$$\begin{cases} \eta(k+1) = \bar{A}\eta(k) + \Delta(k)\bar{B}\bar{f}(k), \\ z(k) = \bar{C}\eta(k), \end{cases} \quad (37)$$

where

$$\bar{A} = \begin{bmatrix} A & BC_c \\ B_cC_1 & A_c \end{bmatrix}, \bar{B} = \begin{bmatrix} B & \mathbf{0} \\ \mathbf{0} & B_c \end{bmatrix}, \bar{C} = [C_2 \ \mathbf{0}],$$
$$\bar{f}(k) = \begin{bmatrix} f_a(k) \\ f_s(k) \end{bmatrix}, \Delta(k) = \begin{bmatrix} \delta_a(k) I & \mathbf{0} \\ \mathbf{0} & \delta_s(k) I \end{bmatrix}, I \in \mathbf{R}^{n\times n}. \quad (38)$$

Then, the control objective is to design a dynamic controller of the form (4) such that

(1) in the condition of $\Delta(k) = \mathbf{0}$, the closed-loop system (37) is asymptotically stable; and
(2) under the zero-initial condition, the controlled output $z(k)$ satisfies

$$\sum_{k=0}^{\infty} \mathbb{E}\{\|\mathbf{z(k)}\|^2\} < \gamma_{\text{both}}^2 \sum_{\mathbf{k=0}}^{\infty} \mathbb{E}\{\|\Delta(\mathbf{k})\tilde{\mathbf{f}}(\mathbf{k})\|^2\}. \quad (39)$$

Theorem 5: *For a prescribed scalar $\gamma_{\text{both}} > 0$ and controller parameters A_c, B_c, and C_c, if there exists a positive-definite matrix $P = P^T$ such that the following matrix inequality constraint holds:*

$$\begin{bmatrix} -P & \mathbf{0} & P\bar{A} & P\bar{B}_1 \\ * & -I & \bar{C} & \mathbf{0} \\ * & * & -P & \mathbf{0} \\ * & * & * & -\gamma_{both}^2\bar{\Delta} \end{bmatrix} < 0, \quad (40)$$

where

$$\bar{B}_1 = \mathbb{E}\{\Delta(k)\}\bar{B} = \begin{bmatrix} \beta B & \mathbf{0} \\ \mathbf{0} & \alpha B_c \end{bmatrix}, \bar{\Delta} = \mathbb{E}\left\{\Delta^T(k)\Delta(k)\right\}$$
$$= \begin{bmatrix} \beta I & \mathbf{0} \\ \mathbf{0} & \alpha I \end{bmatrix}, I \in \mathbf{R}^{n\times n},$$

then the control objective is satisfied.

Proof: From inequality (40), the following constraint is inferred by using the Schur complement lemma:

$$\begin{bmatrix} \bar{A}^T P\bar{A} - P + \bar{C}^T\bar{C} & \bar{A}^T P\bar{B}_1 \\ \bar{B}_1^T P\bar{A} & \bar{B}_1^T P\bar{B}_1 - \gamma_{\text{both}}^2\bar{\Delta} \end{bmatrix} < 0, \quad (41)$$

which implies $\bar{A}^T P\bar{A} - P < 0$. Hence, with a Lyapunov function defined the same way as (14), it is obvious that the difference of the Lyapunov function $\Delta V(k)$ is negative under the condition of $\Delta(k) = \mathbf{0}$. Then, the closed-loop system (37) is asymptotically stable when $\Delta(k) = \mathbf{0}$.

Similar to the steps in the proof of Theorem 1, the following inequality is derived based on (41):

$$\mathbb{E}\{V(\eta(k+1))\} - \mathbb{E}\{V(\eta(k))\} + \mathbb{E}\left\{z^T(k)z(k)\right\} - \gamma_{both}^2 \mathbb{E}\left\{\bar{f}^T(k)\Delta^T(k)\Delta(k)\bar{f}(k)\right\} < 0. \quad (42)$$

By summing inequality (42) from $k = 0$ to ∞, the control performance index (39) is inferred under the zero-initial condition which completes the proof. □

Theorem 6: *For a prescribed scalar $\gamma_{\text{both}} > 0$, if there exist positive-definite matrices $X = X^T$, $Y = Y^T$, and real matrices $\hat{A}$, $\hat{B}$, and $\hat{C}$ such that the following LMI (43) holds:*

$$\begin{bmatrix} -X & -I & \mathbf{0} & AX + B\hat{C} & A & \beta B & \mathbf{0} \\ * & -Y & \mathbf{0} & \hat{A} & YA + \hat{B}C_1 & \beta YB & \alpha\hat{B} \\ * & * & -I & C_2X & C_2 & \mathbf{0} & \mathbf{0} \\ * & * & * & -X & -I & \mathbf{0} & \mathbf{0} \\ * & * & * & * & -Y & \mathbf{0} & \mathbf{0} \\ * & * & * & * & * & -\beta\gamma_{\text{both}}^2 I & \mathbf{0} \\ * & * & * & * & * & * & -\alpha\gamma_{\text{both}}^2 I \end{bmatrix} < 0. \quad (43)$$

Then, the closed-loop system (37) is asymptotically stable in the condition of $\Delta(k) = \mathbf{0}$, and inequality (39) can be satisfied under zero-initial condition. Moreover, the controller parameters are determined by (20) and (21).

Proof: With the variables P, P^{-1}, Π_1, Π_2, $\hat{A}$, $\hat{B}$, and $\hat{C}$ defined in (22)–(25) and the equivalent connections described by (26), conducting the congruence transformation by diag$\left(\prod_1, I, \prod_1, I\right)$ to (40) leads to the LMI (43). Similar to the proof of Theorem 2, controller parameters (20) are obtained from (25). This completes the proof. □

4. An illustrative example

In this section, a numerical example is taken to demonstrate the effectiveness and applicability of the proposed method under random intermittent faults. The system matrices of (1) are defined as follows:

$$A = \begin{bmatrix} 1.1 & 0.1 \\ 0.3 & 0.2 \end{bmatrix}, \; B = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}.$$

The occurrence probabilities of the sensor fault and the actuator fault are $\alpha = \beta = 0.02$. Let the prescribed H_∞ performance level $\gamma_s = 5$, $\gamma_a = 3$, and $\gamma_{\text{both}} = 3$. By using the LMI ToolBox, the corresponding controllers' parameters of the three cases are obtained.

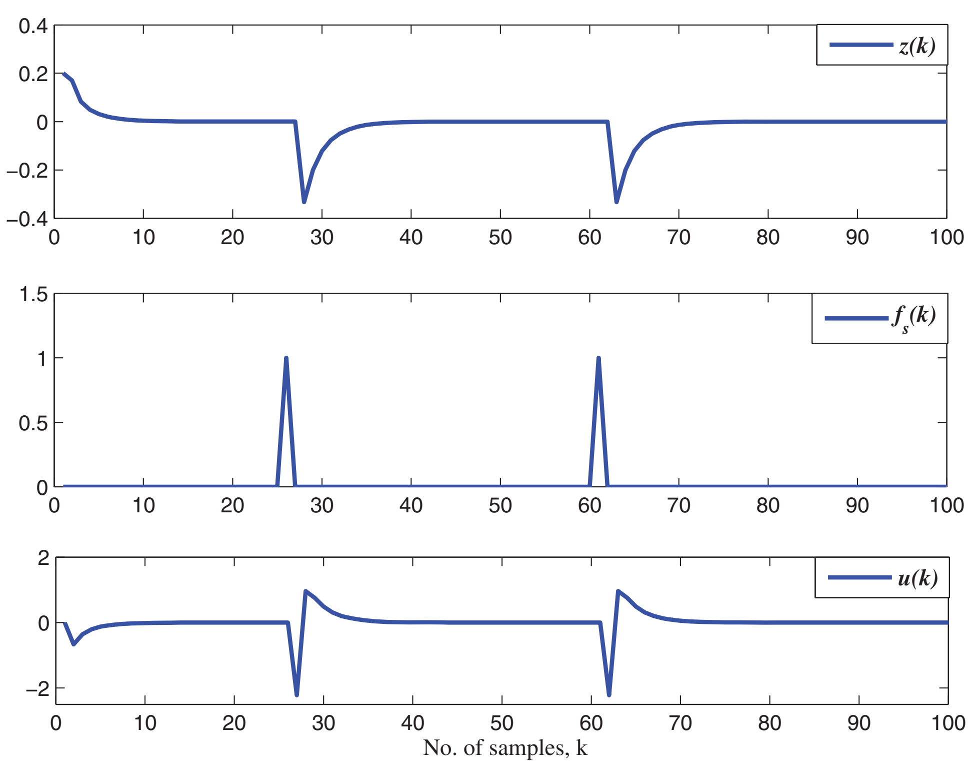


Figure 1. The controlled output $z(k)$, the sensor fault $f_s(k)$, and the controller output $u(k)$.

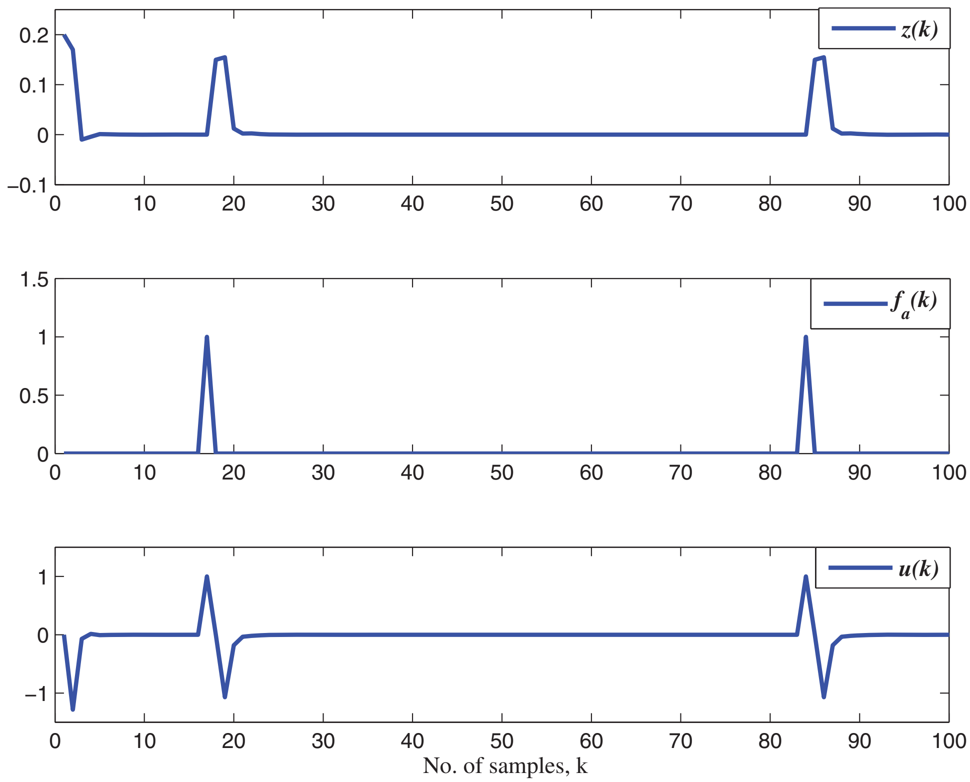


Figure 2. The controlled output $z(k)$, the actuator fault $f_a(k)$, and the controller output $u(k)$.

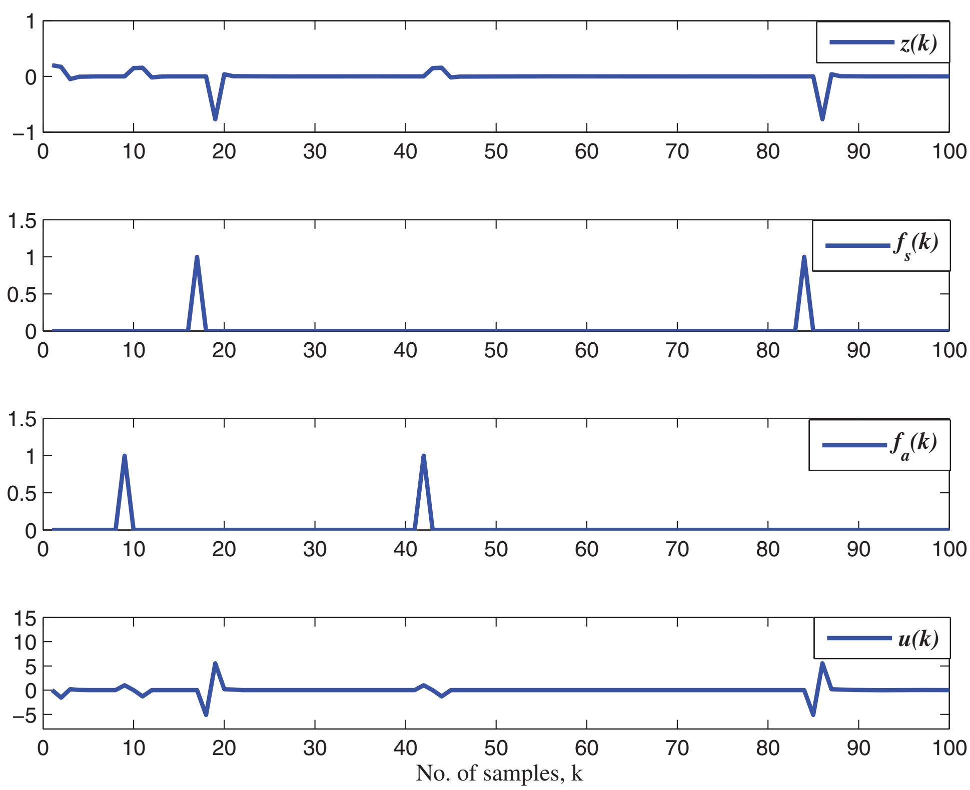


Figure 3. The controlled output $z(k)$, the two faults, and the controller output $u(k)$.

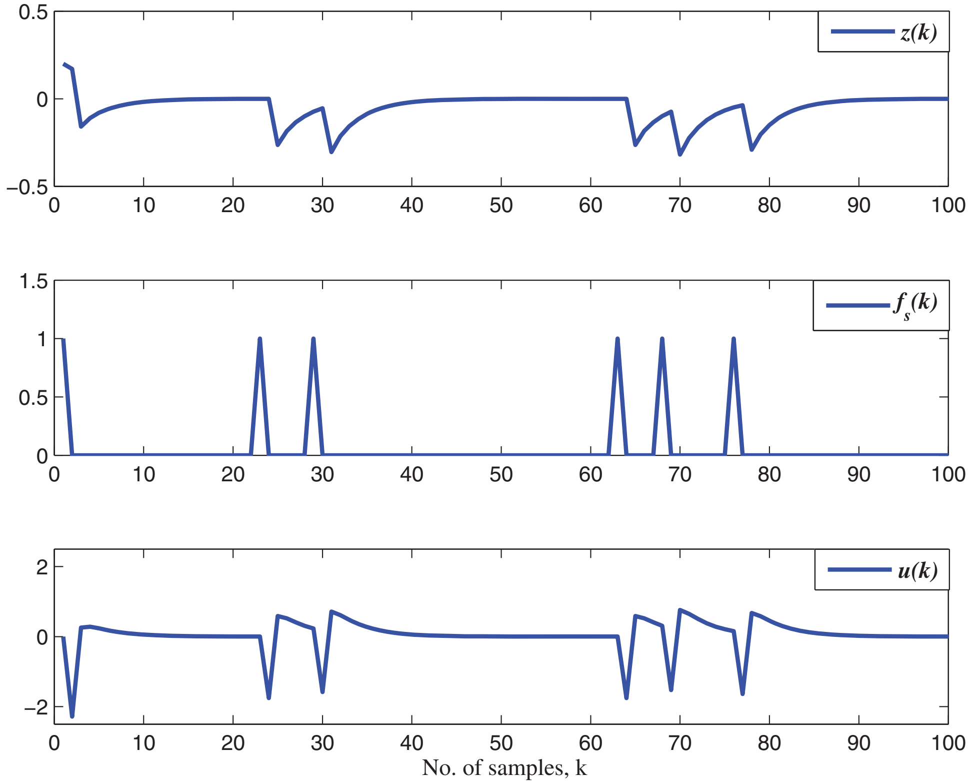


Figure 4. The controlled output $z(k)$, the sensor fault $f_s(k)$, and the controller output $u(k)$.

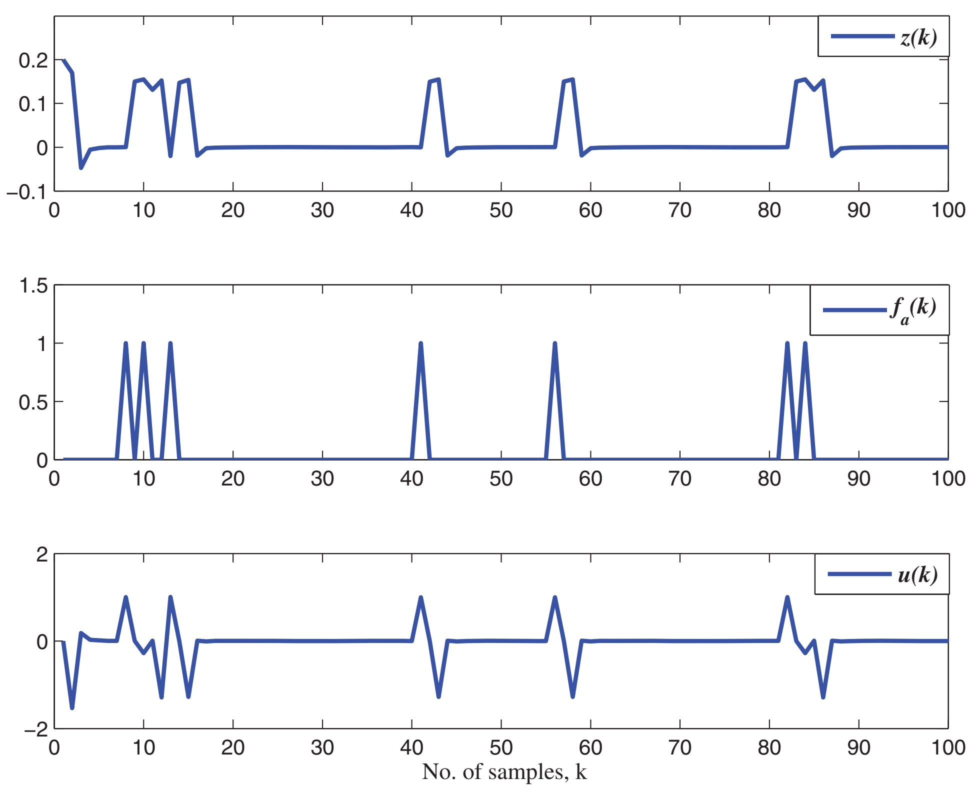


Figure 5. The controlled output $z(k)$, the actuator fault $f_a(k)$ and the controller output $u(k)$.

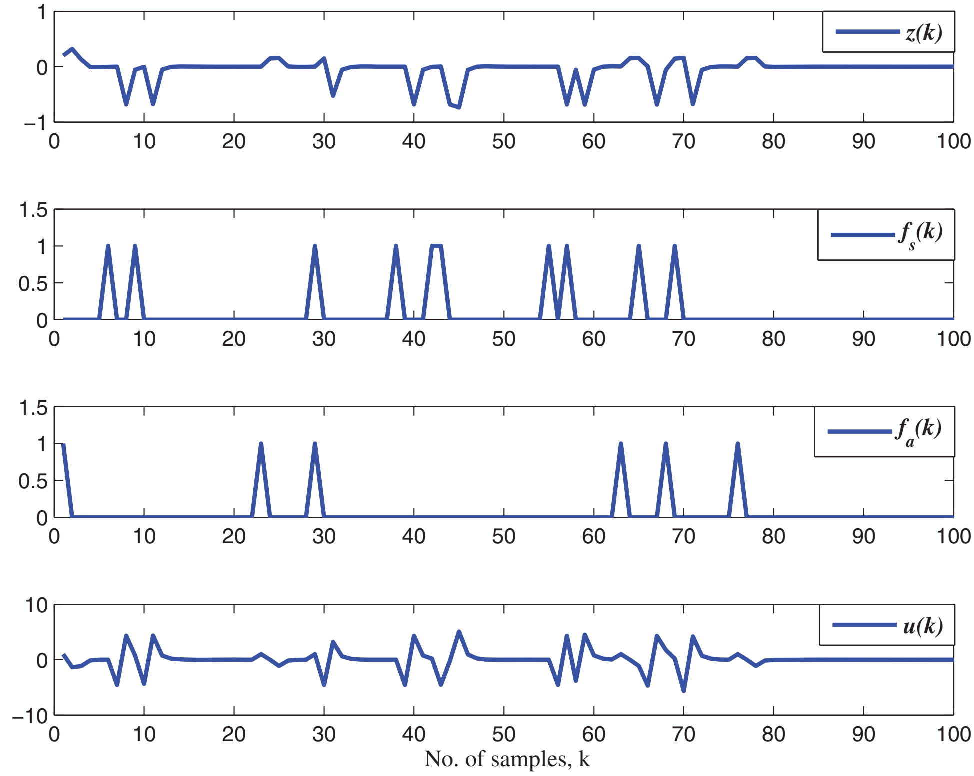


Figure 6. The controlled output $z(k)$, the two faults, and the controller output $u(k)$.

For Case A,

$$A_c = \begin{bmatrix} 0.1816 & -0.0176 \\ 0.0079 & -0.4748 \end{bmatrix}, \; B_c = \begin{bmatrix} -0.1910 \\ 1.0213 \end{bmatrix},$$
$$C_c = \begin{bmatrix} 0.6921 & -2.0449 \end{bmatrix}.$$

For Case B,

$$A_c = \begin{bmatrix} -0.4561 & -0.5548 \\ -0.5724 & -0.3116 \end{bmatrix}, \; B_c = \begin{bmatrix} 1.2071 \\ 1.3675 \end{bmatrix},$$
$$C_c = \begin{bmatrix} -1.2560 & -2.0254 \end{bmatrix}.$$

For Case C,

$$A_c = \begin{bmatrix} -0.2468 & -0.6182 \\ -0.6896 & -0.6569 \end{bmatrix}, \; B_c = \begin{bmatrix} 0.5428 \\ 1.0509 \end{bmatrix},$$
$$C_c = \begin{bmatrix} -1.5086 & -4.0925 \end{bmatrix}.$$

The simulation results are shown in Figures 1–3. In all these simulations, since the sensor and actuator of the considered system have only one dimension, it is assumed that the amplitudes of the sensor fault and the actuator fault equal to 1. Obviously, simulation results illustrate that the reliable controllers can maintain the closed-loop system stable even if random intermittent faults on sensors and actuators occur in terms of the three above-mentioned cases. Taking Figure 1 as an example, the reliable controller could stabilise the system rapidly. Meanwhile, after the advent of a sensor fault, although the controlled output shows an evident vibration, it converges to zero within several steps.

Furthermore, an additional simulation based on the same system model is conducted with higher occurrence rates ($\alpha = \beta = 0.1$) of both considered faults to certify the performance of the designed method. Figures 4–6 show that even with a higher possibility of the presence of faults, the reliable controllers could stabilise the relevant systems efficiently.

5. Conclusions

This paper has proposed a reliable H_∞ controller design strategy for discrete-time systems against random intermittent faults which may take place at the sensor side and/or the actuator side. The intermittent faults are governed by random variables which obey the Bernoulli distribution. Sufficient conditions of the existence of reliable controllers with regard to three considered fault-occurring cases are developed by LMI. Under the proposed passive fault-tolerant control scheme, the closed-loop systems could achieve an acceptable performance even if random intermittent faults happen. An illustrative example demonstrates the effectiveness of the proposed method.

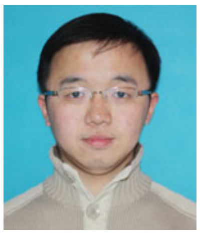
Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work was supported by National Natural Science Foundation of China [grant number 61374099]; Program for New Century Excellent Talents in University [grant number NCET-13-0652]; The Scientific Research Foundation for the Returned Overseas Chinese Scholars, State Education Ministry [grant number LXJJ201301].

Notes on contributors



Yuan Tao received his BS degree from Beijing University of Chemical Technology, Beijing, China, in 2008, where he is currently working towards his PhD degree in control theory and control engineering. His research interests include fault-tolerant control, robust control, and anaesthesia closed-loop control.

Dong Shen received his BS degree in mathematics from Shandong University, Jinan, China, in 2005. He received his PhD degree in mathematics from the Academy of Mathematics and System Science, Chinese Academy of Sciences (CAS), Beijing, China, in 2010. From 2010 to 2012, he was a post-doctoral fellow with the Institute of Automation, CAS. Since 2012, he has been an associate professor with the College of Information Science and Technology, Beijing University of Chemical Technology (BUCT), Beijing, China. His current research interests include iterative learning controls, stochastic control, and optimisation.

Mengqi Fang received her BS degree from Beijing University of Chemical Technology, Beijing, China, in 2012, where she is currently working towards her Master's degree in the Control Theory and Control Engineering. Her research interests include subspace-based system identification, model predictive control, and anaesthesia closed-loop control.

Youqing Wang received his BS degree from Shandong University, Jinan, Shandong, China, in 2003, and his PhD degree in control science and engineering from Tsinghua University, Beijing, China, in 2008. He worked as a research assistant in the Department of Chemical Engineering, Hong Kong University of Science and Technology, from February 2006 to August 2007. From February 2008 to February 2010, he worked as a senior investigator in the Department of Chemical Engineering, University of California, Santa Barbara, CA, USA. From March 2008 to December 2011, he was an adjunct senior investigator in Sansum Diabetes Research Institute. Currently, he is a full professor in Beijing University of Chemical Technology. His research interests include fault-tolerant control, state monitoring, modelling and control of biomedical processes (e.g. artificial pancreas system), and iterative learning control. He is an (associate) editor of *Multidimensional Systems and Signal Processing* and *Mathematical Problems in Engineering*. He is also a member of the IFAC Technical Committee on Biological and Medical Systems. He is a recipient of several research awards (including Journal of Process Control Survey Paper Prize and Zhang Zhong-Jun Academician Outstanding Paper Award).

References

Chilin, D., Liu, J., Davis, J.F., & Christofides, P.D. (2012). Data-based monitoring and reconfiguration of a distributed model predictive control system. *International Journal of Robust and Nonlinear Control, 22*, 68–88.

Correcher, A., Garcia, E., Morant, F., Quiles, E., & Rodriguez, L. (2012). Intermittent failure dynamics characterization. *IEEE Transactions on Reliability, 61*, 649–658.

Dong, Q., Zhong, M., & Ding, S.X. (2012). Active fault tolerant control for a class of linear time-delay systems in finite frequency domain. *International Journal of Systems Science, 43*, 543–551.

Gao, Z., Jiang, B., Qi, R., & Xu, Y. (2011). Robust reliable control for a near space vehicle with parametric uncertainties and actuator faults. *International Journal of Systems Science, 42*, 2113–2124.

Gassara, H., El Hajjaji, A., Kchaou, M., & Chaabane, M. (2014). Robust H_∞ reliable control of time delay nonlinear systems via Takagi–Sugeno fuzzy models. *International Journal of Systems Science, 45*, 667–681.

Gu, Z., Zhang, J., & Du, L. (2011). Fault tolerant control for a class of time-delay systems with intermittent actuators failure. *Control and Decision, 26*, 1829–1834.

Guenab, F., Weber, P., Theilliol, D., & Zhang, Y.M. (2011). Design of a fault tolerant control system incorporating reliability analysis and dynamic behaviour constraints. *International Journal of Systems Science, 42*, 219–233.

He, X., Hu, Y., & Peng, K. (2013). Intermittent fault detection for uncertain networked systems. *Mathematical Problems in Engineering, 2013*, 1–10.

He, X., Wang, Z., Ji, Y.D., & Zhou, D.H. (2011). Robust fault detection for networked systems with distributed sensors. *IEEE Transactions on Aerospace and Electronic Systems, 47*, 166–177.

He, X., Wang, Z., Liu, Y., & Zhou, D.H. (2013). Least-squares fault detection and diagnosis for networked sensing systems using a direct state estimation approach. *IEEE Transactions on Industrial Informatics, 9*, 1670–1679.

Hu, Q., & Xiao, B. (2013). Adaptive fault tolerant control using integral sliding mode strategy with application to flexible spacecraft. *International Journal of Systems Science, 44*, 2273–2286.

Hua, C.-C., Ding, S.X., & Guan, X.-P. (2012). Robust controller design for uncertain multiple-delay systems with unknown actuator parameters. *Automatica, 48*, 211–218.

Hwang, I., Kim, S., Kim, Y., & Seah, C.E. (2010). A survey of fault detection, isolation, and reconfiguration methods. *IEEE Transactions on Control Systems Technology, 18*, 636–653.

Kargar, S.M., Salahshoor, K., & Yazdanpanah, M.J. (2014). Integrated nonlinear model predictive fault tolerant control and multiple model based fault detection and diagnosis. *Chemical Engineering Research & Design, 92*, 340–349.

Liu, Y., Wang, Z., & Wang, W. (2011). Reliable H_∞ control for discrete uncertain time-delay systems with randomly occurring nonlinearities: The output feedback case. *International Journal of Systems Science, 42*, 809–820.

Mahmoud, M., Jiang, J., & Zhang, Y.M. (2001). Stochastic stability analysis of fault-tolerant control systems in the presence of noise. *IEEE Transactions on Automatic Control, 46*, 1810–1815.

Scherer, C., Gahinet, P., & Chilali, M. (1997). Multiobjective output-feedback control via LMI optimization. *IEEE Transactions on Automatic Control, 42*, 896–911.

Seron, M.M., De Dona, J.A., & Olaru, S. (2012). Fault tolerant control allowing sensor healthy-to-faulty and faulty-to-healthy transitions. *IEEE Transactions on Automatic Control, 57*, 1657–1669.

Tabatabaeipour, S.M., Izadi-Zamanabadi, R., Bak, T., & Ravn, A.P. (2012). Passive fault-tolerant control of discrete time piecewise affine systems against actuator faults. *International Journal of Systems Science, 43*, 1985–1997.

Tian, E., Yue, D., & Peng, C. (2010). Reliable control for networked control systems with probabilistic sensors and actuators faults. *IET Control Theory and Applications, 4*, 1478–1488.

Wang, H., & Ju, H.-H. (2013). Reliable H filtering for LPV systems with sensor faults in finite frequency domain. *International Journal of Systems Science, 44*, 2310–2320.

Wang, Y., Shi, J., Zhou, D., & Gao, F. (2006). Iterative learning fault-tolerant control for batch processes. *Industrial & Engineering Chemistry Research, 45*, 9050–9060.

Wang, Y., Zhou, D., & Gao, F. (2007). Robust fault-tolerant control of a class of non-minimum phase nonlinear processes. *Journal of Process Control, 17*, 523–537.

Wang, Z., Shen, B., Shu, H., & Wei, G. (2012). Quantized H_∞ control for nonlinear stochastic time-delay systems with missing measurements. *IEEE Transactions on Automatic Control, 57*, 1431–1444.

Witczak, M., Puig, V., & Montes de Oca, S. (2013). A fault-tolerant control strategy for non-linear discrete-time systems: Application to the twin-rotor system. *International Journal of Control, 86*, 1788–1799.

Xu, Q., Yang, H., Jiang, B., Zhou, D., & Zhang, Y. (2014). Fault tolerant formations control of UAVs subject to permanent and intermittent faults. *Journal of Intelligent & Robotic Systems, 73*, 589–602.

Yang, H., Jiang, B., & Zhang, Y. (2012). Tolerance of intermittent faults in spacecraft attitude control: Switched system approach. *IET Control Theory and Applications, 6*, 2049–2056.

Yao, L., Qin, J., Wang, H., & Jiang, B. (2012). Design of new fault diagnosis and fault tolerant control scheme for non-Gaussian singular stochastic distribution systems. *Automatica, 48*, 2305–2313.

Zhang, Y., & Jiang, J. (2008). Bibliographical review on reconfigurable fault-tolerant control systems. *Annual Reviews in Control, 32*, 229–252.

Zhou, D., Shi, J., & He, X. (2014). Review of intermittent fault diagnosis techniques for dynamic systems. *Acta Automatica Sinica, 40*, 161–171.