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Zero-error convergence of iterative learning control using quantized error information

YUN XU AND DONG SHEN*
College of Information Science and Technology, Beijing University of Chemical Technology, Beijing 100029, People's Republic of China
*Corresponding author: shendong@mail.buct.edu.cn

AND

XUHUI BU
School of Electrical Engineering and Automation, Henan Polytechnic University, Jiaozuo 454000, People's Republic of China

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An iterative learning control algorithm using quantized error information is proposed in this article for both linear and nonlinear systems. The actual output is first compared with the reference signal and then the corresponding error is quantized and transmitted. A logarithmic quantizer is used to guarantee an adaptive improvement for tracking performance. The tracking error under this scheme is proved to converge to zero asymptotically. Illustrative examples verify the theoretical results.

Keywords: iterative learning control; quantized error information; zero-error convergence.

1. Introduction

Iterative learning control (ILC), first proposed in Arimoto *et al.* (1984), is an important branch of intelligent control methods, which aims to improve the tracking performance by successively correcting the input signal from iteration to iteration (Bristow *et al.*, 2006; Ahn *et al.*, 2007; Shen & Wang, 2014). It is inspired by the intuitive idea that one could learn from previous experiences and lessons when conducting some task repeatedly, so that he/she would do it better and better. Thus, ILC is suitable for the systems that could accomplish some tasks over a fixed time interval repetitively. For such systems, the input signal for the current cycle can be formulated using the input and output information of past cycles as well as the tracking objective. As has been developed for three decades, ILC has gained extensive developments both in theory and in applications. Different kinds of systems have been covered, such as networked control systems (Shen & Wang, 2015a,b), multi-agent systems (Meng *et al.*, 2013, 2014a,b, 2015; Meng & Moore, 2016), switched systems (Bu *et al.*, 2013), etc. Some potential industrial applications are also reported such as wind turbines (Tutty *et al.*, 2014), robot manipulator (Zhao *et al.*, 2015), automated off-highway vehicle (Liu & Alleyne, 2014), etc.

Recently more and more practical applications employ the networked control system scheme, where the plant and the controller are usually located in different sites and communicate with each other through wire/wireless networks. In such settings, the ILC under random data dropout has attained a lot of research (Bu *et al.*, 2014; Shen & Wang, 2015a,b). Meanwhile, multi-agent system is also a hot topic in the control society (Meng *et al.*, 2015; Meng & Moore, 2016), where the data communication among agents is a common highlight. In both networked control systems and multi-agent systems, the transmission burden is a critical issue to be addressed for practical applications. In order to reduce the transmission burden, quantization is a potential alternative. Quantized estimation and control have been studied in Curry (1970) and some excellent progress in identification based on quantized observations are reported in Wang *et al.* (2010). A survey on quantized nonlinear control is provided in Jiang & Liu (2013). Brockett & Liberzon (2000) and Fagnani & Zampieri (2004) gave some results on quantized feedback stabilization problems, where Brockett & Liberzon (2000) considered a simple uniform quantizer with saturation, while Fagnani & Zampieri (2004) addressed both uniform quantizer and nested quantizer and made efforts on the tradeoff between quantization complexity and system performance. However, in the ILC field, no paper has been found on quantized control except Bu *et al.* (2015).

Bu *et al.* made the first attempt on ILC problem in Bu *et al.* (2015), where the output measurements were quantized by a logarithmic quantizer and fed to the controller for updating ILC law. By using sector bound method and conventional contraction mapping method, it was shown that the tracking error converged to a small range whose upper bound depended on quantization density. However, the tracking error also depended on the target value, which can be seen from the expression of the upper bound. That is, the larger the output measurement is, the larger the final tracking error bound is. This is natural because of the definition of the quantizer. Thus, in order to get better tracking performance for a large target value, the quantizer density should be much greater. Nevertheless, is there any possible way to make a zero-convergence of ILC with quantized information? This is the problem that the paper addresses.

In this paper, we only consider the case that transmission from system output to controller is quantized for saving network bandwidth. This reason is to make our idea more intuitive, and the general case is left for further study. Motivated by the basic principle of ILC, which is learning and improving iteration by iteration for a specified reference, we make an alteration to the conventional control implementation. To be specific, we first transmit the reference to the system before running the system for a certain task. Then, the system would make a comparison between its output and the given reference and then quantize the error locally. It is the quantized error that is transmitted back to the controller for successive updating. As one could see in the following of this paper, the quantization error reduces adaptively as the tracking error reduces even a sparse logarithmic quantizer is adopted. Moreover, the delivery of the desired reference might be a controversial point of our scheme. However, in the current implementation, the network from the controller to the plant is assumed to work well. In other words, the input signal is well delivered. Therefore, we could use this network to transmit the accurate reference.

The rest of the paper is arranged as follows: Section 2 provides the system formulation and problem statements; while main results and analysis are given in Section 3; the extension to nonlinear systems is shown in Section 4; Section 5 provides some illustrative simulations to verify the theoretical analysis; and Section 6 concludes this paper.

2. Problem formulation

Consider the following linear discrete-time system

$$\begin{aligned} x_k(t+1) &= Ax_k(t) + Bu_k(t), \\ y_k(t) &= Cx_k(t), \end{aligned} \tag{2.1}$$

where $k = 1, 2, \ldots$ denotes the number of different iterations and $t = 0, 1, \ldots, N$ denotes different time instances in an iteration. Here N is the iteration length. $x_k(t)$, $u_k(t)$ and $y_k(t)$ are state, input and output, respectively. A, B and C are suitable matrices with appropriate dimensions. Without loss of any generality, it is assumed that CB is of full-column rank.

The reference is denoted by $y_d(t)$, $t = \{0, 1, \ldots, N\}$. The control objective is to find an input sequence $\{u_k(t)\}$ such that the output $y_k(t)$ would converge to $y_d(t)$ as $k \to \infty$, $\forall t$. For further analysis, the following assumptions are needed.

ASSUMPTION 2.1 The reference $y_d(t)$ is realizable, i.e. there is an unique input $u_d(t)$ such that

$$\begin{aligned} x_d(t+1) &= Ax_d(t) + Bu_d(t), \\ y_d(t) &= Cx_d(t), \end{aligned} \tag{2.2}$$

with a suitable initial state $x_d(0)$.

ASSUMPTION 2.2 The identical initial condition is satisfied, i.e. for all iterations,

$$x_k(0) = x_d(0), \tag{2.3}$$

where $x_d(0)$ is the desired initial state.

In this paper, for any specified tracking reference, we first transmit it to the system before operating. Then the tracking error is generated, quantized and transmitted back to controller. In other words, the ILC law is

$$u_{k+1}(t) = u_k(t) + LQ(y_d(t+1) - y_k(t+1)), \tag{2.4}$$

where L is the learning gain matrix while $Q(\cdot)$ is a selected quantizer. In this paper, a logarithmic quantizer as in Bu *et al.* (2015) and Elia & Mitter (2001) is adopted,

$$\begin{aligned} &U = \{\pm z_i : z_i = \mu^i z_0, i = 0, \pm 1, \pm 2, \cdots\} \cup \{0\}, \\ &0 < \mu < 1, z_0 > 0, \end{aligned} \tag{2.5}$$

where μ is associated with the quantization density. The associated quantizer $Q(\cdot)$ is given as

$$Q(v) = \begin{cases} z_i, & \text{if } \frac{1}{1+\zeta} z_i < v \le \frac{1}{1-\zeta} z_i, \\ 0, & \text{if } v = 0, \\ -Q(-v), & \text{if } v < 0, \end{cases} \tag{2.6}$$

with $\zeta = (1-\mu)/(1+\mu)$. It is evident that the quantizer $Q(\cdot)$ in (2.6) is symmetric and time-invariant.

Denote the tracking error $e_k(t) = y_d(t) - y_k(t)$. In Fu & Xie (2005), a sector bound method is proposed to deal with the quantization error. In this article, for a given quantization density μ, we have

$$Q(e_k(t)) - e_k(t) = \Delta e_k(t) \cdot e_k(t), \tag{2.7}$$

where $|\Delta e_k(t)| \le \zeta$ and $\zeta = (1-\mu)/(1+\mu)$.

REMARK 1 One may ask whether the quantization error can be regarded as a disturbance or uncertainty and the robust control analysis techniques can then be applied to derive some results. In our opinion, this is an alternative approach to deal with the quantization error. However, this formulation of the quantization error would lead to conservative results regarding the tracking error's convergence. That is, the magnitude is not clear enough if the error is modelled as a disturbance or uncertainty. Meanwhile, following the robust control analysis techniques, one could only prove that the tracking error would converge into a bounded range. In this paper, we would like to derive more insightful convergence.

3. Main results

In this section, the zero-error convergence of (2.4) is given in the following theorem.

THEOREM 3.1 Consider system (2.1) and update law (2.4) with quantized error, and assume A2.1–A2.2 hold. If L is designed such that

$$\|I - LCB\| + \zeta \|LCB\| \le \lambda < 1, \tag{3.1}$$

then the system tracking error converges to zero as $k \to \infty$.

Proof. Denote $\delta u_k(t) = u_d(t) - u_k(t)$, $\delta x_k(t) = x_d(t) - x_k(t)$. Subtracting both sides of (2.4) from $u_d(t)$, and combining (2.7), we have

$$\begin{aligned} \delta u_{k+1}(t) =& \delta u_k(t) - LQ(e_k(t+1)) \\ =& \delta u_k(t) - L(e_k(t+1) + \Delta e_k(t+1) \cdot e_k(t+1)) \\ =& \delta u_k(t) - L(C\delta x_k(t+1) + \Delta e_k(t+1) \cdot C\delta x_k(t+1)). \end{aligned} \tag{3.2}$$

From system (2.1) and A2.1, one has

$$\begin{aligned} \delta x_k(t+1) =& x_d(t+1) - x_k(t+1) \\ =& Ax_d(t) + Bu_d(t) - Ax_k(t) - Bu_k(t) \\ =& A\delta x_k(t) + B\delta u_k(t). \end{aligned} \tag{3.3}$$

Thus, it is evident that

$$\begin{aligned} \delta u_{k+1}(t) =& (I - LCB)\delta u_k(t) - L\Delta e_k(t+1)CB\delta u_k(t) \\ & - (LCA\delta x_k(t) + L\Delta e_k(t+1)CA\delta x_k(t)) \\ =& (I - LCB)\delta u_k(t) - \Delta e_k(t+1)LCB\delta u_k(t) \\ & - (1 + \Delta e_k(t+1))LCA\delta x_k(t). \end{aligned} \tag{3.4}$$

Taking norms of both sides of last equation leads to

$$\begin{aligned} \|\delta u_{k+1}(t)\| \le& \|I - LCB\|\|\delta u_k(t)\| + \zeta\|LCB\|\|\delta u_k(t)\| \\ & + (1+\zeta)\|LCA\|\|\delta x_k(t)\| \\ \le& \lambda\|\delta u_k(t)\| + \theta\|\delta x_k(t)\|, \end{aligned} \tag{3.5}$$

where $\|I - LCB\| + \zeta\|LCB\| \le \lambda$, $(1+\zeta)\|LCA\| = \theta$. On the other hand, taking norms of (3.3) yields

$$\|\delta x_k(t+1)\| \le \|A\|\|\delta x_k(t)\| + \|B\|\|\delta u_k(t)\|. \tag{3.6}$$

Recursively, we have

$$\begin{aligned}\|\delta x_k(t)\| &\le \|A\|\|\delta x_k(t-1)\| + \|B\|\|\delta u_k(t-1)\| \\ &\le \|A\|^2\|\delta x_k(t-2)\| + \|A\|\|B\|\|\delta u_k(t-2)\| + \|B\|\|\delta u_k(t-1)\| \\ &\le \sum_{i=0}^{t-1} \|A\|^{t-1-i}\|B\|\|\delta u(i)\|,\end{aligned} \tag{3.7}$$

where the last inequality uses the assumption A2.2. Combining (3.5) and (3.7) we have

$$\|\delta u_{k+1}(t)\| \le \lambda\|\delta u_k(t)\| + \theta\sum_{i=0}^{t-1}\|A\|^{t-1-i}\|B\|\|\delta u(i)\|. \tag{3.8}$$

Specifically,

$$\begin{aligned}\|\delta u_{k+1}(0)\| &\le \lambda\|\delta u_k(0)\| \\ \|\delta u_{k+1}(1)\| &\le \lambda\|\delta u_k(1)\| + \theta\|B\|\|\delta u(0)\| \\ &\cdots \\ \|\delta u_{k+1}(N-1)\| &\le \lambda\|\delta u_k(N-1)\| + \theta\sum_{i=0}^{N-2}\|A\|^{N-2-i}\|B\|\|\delta u(i)\|.\end{aligned}$$

Denote $\delta U_k = [\|\delta u_k(0)\|, \|\delta u_k(1)\|, \cdots, \|\delta u_k(N-1)\|]^{\mathrm{T}}$. It is observed that

$$\delta U_{k+1} \le \Gamma\delta U_k, \tag{3.9}$$

where

$$\Gamma = \begin{bmatrix} \lambda & 0 & \cdots & 0 \\ \theta\|B\| & \lambda & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \theta\|A\|^{N-2}\|B\| & \theta\|A\|^{N-3}\|B\| & \cdots & \lambda \end{bmatrix}. \tag{3.10}$$

Notice that Γ is a lower triangular system with diagonal elements being $\lambda < 1$, thus, $\delta U_k \xrightarrow[k\to\infty]{} 0$, or equivalently, $\|\delta u_k(t)\| \xrightarrow[k\to\infty]{} 0$, $\forall t$.

Noticing (3.7) and the finiteness of time, we have $\|\delta x_k(t)\| \xrightarrow[k\to\infty]{} 0$, $\forall t$. This further yields that $\|e_k(t)\| \xrightarrow[k\to\infty]{} 0$, $\forall t$. In other words, a zero-error tracking performance of the system is obtained. This completes the proof. □

REMARK 1 As can be seen from (2.7), the zero-error convergence of tracking performance is guaranteed because the quantization error is bounded by the actual tracking error. Consequently, larger tracking error means larger quantization error. However, it is acceptable since the learning could be rough at this stage. While as the tracking error converges to zero, the quantization error also reduces to zero and thus enables a zero-error tracking as the iteration number goes to infinity.

REMARK 2 Notice that the quantization error is bounded by tracking error, thus it leaves much freedom for the design of quantizer which is characterized by ζ. However, as explained in Bu *et al.* (2015), quantizer index ζ and learning gain matrix L are coupled with each other in the convergence condition (3.1). Thus, the selection of quantizer is not totally free.

REMARK 3 The asymptotic convergence to zero is proved in the theorem. One may be interested in a monotonic convergence that ensures a good transient performance along iterations. Noticing (3.10), it is observed that a contraction on $\|\delta U_k\|$ depends on the property of Γ such as $\|\Gamma\| < 1$ where $\|\delta U_k\|$ and $\|\Gamma\|$ denote some consistent norms of vector and matrix. Meanwhile, the quantization and the design of learning matrix L should satisfy that $\|I - LCB\| + \zeta\|LCB\| < 1$ to guarantee such monotone convergence.

4. Extension to nonlinear systems

In this section, the following nonlinear system is considered

$$\begin{aligned} x_k(t+1) &= f(x_k(t)) + B(x_k(t))u_k(t), \\ y_k(t) &= Cx_k(t), \end{aligned} \tag{4.1}$$

where $f(x_k(t))$ and $B(x_k(t))$ are functions of $x_k(t)$.

The following assumptions are needed for further analysis.

ASSUMPTION 4.1 The reference $y_d(t)$ is realizable, i.e. there is an unique input $u_d(t)$ such that

$$\begin{aligned} x_d(t+1) &= f(x_d(t)) + B(x_d(t))u_d(t), \\ y_d(t) &= Cx_d(t), \end{aligned} \tag{4.2}$$

with a suitable initial state $x_d(0)$.

ASSUMPTION 4.2 The nonlinear functions $f(\cdot)$, $B(\cdot)$ satisfy globally Lipschitz condition in their arguments, i.e. there exist b_f and b_B such that

$$\begin{aligned} \|f(x_1) - f(x_2)\| &\le b_f\|x_1 - x_2\|, \\ \|B(x_1) - B(x_2)\| &\le b_B\|x_1 - x_2\|. \end{aligned}$$

Now the theorem of zero-convergence for nonlinear system is given.

THEOREM 4.1 Consider system (4.1) and update law (2.4) with quantized error, and assume A2.2, A4.1–A4.2 hold. If L is designed such that

$$\|I - LCB(x_k(t))\| + \zeta\|LCB(x_k(t))\| \leq \lambda < 1, \quad \forall k, t, \tag{4.3}$$

then the system tracking error converges to zero as $k \to \infty$.

Proof. The proof follows similar steps of Theorem 3.1 with minor modifications.

The derivation of (3.2) is still valid. Combining (4.1) and (4.2), we have

$$\begin{aligned}\delta x_k(t+1) =& x_d(t+1) - x_k(t+1)\\ =& f(x_d(t)) - f(x_k(t)) + B(x_d(t))u_d(t) - B(x_k(t))u_k(t)\\ =& B(x_k(t))\delta u_k(t) + f(x_d(t)) - f(x_k(t))\\ &+ [B(x_d(t)) - B(x_k(t))]u_d(t).\end{aligned} \tag{4.4}$$

Substitute this equation into (3.2), then we have

$$\begin{aligned}\delta u_{k+1}(t) =& \delta u_k(t) - L(C\delta x_k(t+1) + \Delta e_k(t+1)C\delta x_k(t+1))\\ =& (I - LCB(x_k(t)))\delta u_k(t) - L\Delta e_k(t+1)CB(x_k(t))\delta u_k(t)\\ &- LC[f(x_d(t)) - f(x_k(t))] - LC[B(x_d(t)) - B(x_k(t))]u_d(t)\\ &- L\Delta e_k(t+1)C[f(x_d(t)) - f(x_k(t))]\\ &- L\Delta e_k(t+1)C[B(x_d(t)) - B(x_k(t))]u_d(t).\end{aligned} \tag{4.5}$$

Taking norm of last equation and using A4.2 lead to

$$\begin{aligned}\|\delta u_{k+1}(t)\| \leq& (\|I - LCB(x_k(t))\| + \zeta\|LCB(x_k(t))\|)\|\delta u_k(t)\|\\ &+ (1+\zeta)\|LC\|b_f\|\delta x_k(t)\|\\ &+ (1+\zeta)\|LC\|\|u_d(t)\|b_B\|\delta x_k(t)\|\\ \leq& \lambda\|\delta u_k(t)\| + \eta\|\delta x_k(t)\|,\end{aligned} \tag{4.6}$$

where $\eta = (1+\zeta)\|LC\|b_f + (1+\zeta)\|LC\|\|u_d(t)\|b_B$.

On the other hand, taking norm of both sides of (4.4) and using A4.2, we have

$$\|\delta x_k(t+1)\| \leq b_B\delta u_k(t) + (b_f + b_B\|u_d(t)\|)\delta x_k(t)$$

and recursively, we have

$$\|\delta x_k(t)\| \leq \sum_{i=0}^{t-1} \rho^{t-1-i} b_B \|\delta u_k(i)\|, \tag{4.7}$$

where $\rho = b_f + b_B \sup_t \|u_d(t)\|$ and A2.2 is used. Therefore, substituting (4.7) into (4.6) yields

$$\|\delta u_{k+1}(t)\| \le \lambda\|\delta u_k(t)\| + \sum_{i=0}^{t-1} \eta\rho^{t-1-i} b_B \|\delta u_k(i)\| \tag{4.8}$$

which could be lifted as

$$\delta U_{k+1} \le \Lambda \delta U_k, \tag{4.9}$$

where

$$\Lambda = \begin{bmatrix} \lambda & 0 & \cdots & 0 \\ \eta b_B & \lambda & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \eta\rho^{N-2} b_B & \eta\rho^{N-3} b_B & \cdots & \lambda \end{bmatrix}.$$

Therefore, it is evident that $\|\delta u_k(t)\| \xrightarrow[k\to\infty]{} 0, \forall t$ and further $\|\delta x_k(t)\| \xrightarrow[k\to\infty]{} 0$, $\|e_k(t)\| \xrightarrow[k\to\infty]{} 0, \forall t$. That is, a zero-error tracking performance of the system is obtained. This completes the proof. □

5. Illustrative simulations

In order to make improvements more visual, the examples of Bu *et al.* (2015) are taken into account in this section.

5.1. *Linear system*

The following linear system is considered in Bu *et al.* (2015):

$$\begin{aligned} x_k(t+1) &= \begin{pmatrix} -0.8 & -0.22 \\ 1 & 0 \end{pmatrix} x_k(t) + \begin{pmatrix} 0.5 \\ 1 \end{pmatrix} u_k(t), \\ y_k(t) &= (1 \quad 0.5)\, x_k(t). \end{aligned} \tag{5.1}$$

The desired reference is given as $y_d(t) = \sin(8t/50) + \sin(4t/50)$, $t \in [0, 100]$. The initial states are set to be $x_k(0) = x_d(0) = 0$ for all k, and the initial input is simply chosen as $u_0(t) = 0, \forall t$. The parameters in the quantizer is given as $z_0 = 2$, $\mu = 0.85$, then $\zeta = 0.08$. In addition, the learning gain L is selected to be 0.8 such that $\|I - LCB\| + \zeta\|LCB\| = 0.264 < 1$. The algorithm is performed for 20 iterations.

The tracking performances at the 2nd, 5th and 20th iterations are shown in Fig. 1, where one could find that the tracking at the fifth iteration is good enough and the output at the 20th iteration almost coincides with the reference.

The tracking error along iteration axis is shown in Fig. 2. The algorithm proposed in Bu *et al.* (2015) is also simulated to make a comparison. To be specific, the solid line denotes the algorithm provided in this paper, where the quantizer is added to the tracking error, while the dashed line denotes the one of Bu *et al.* (2015), where the quantizer is added to the actual output. As can be seen from Fig. 2, the maximum tracking error of the algorithm in Bu *et al.* (2015) could not reduce to zero due to quantization error. However, the update law in this paper ensures a zero-error convergence.

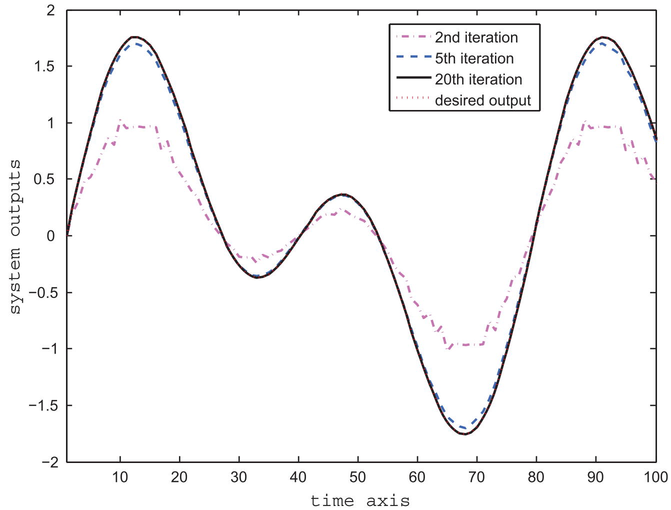

FIG. 1. Outputs at different iterations vs reference.

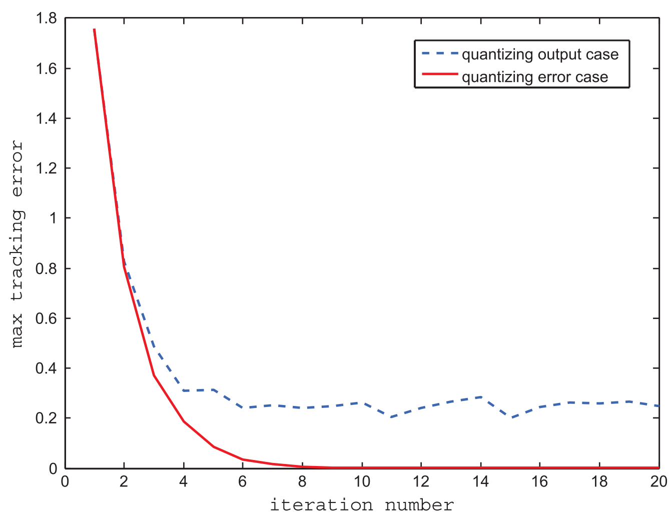

FIG. 2. Maximal tracking error along iteration axis.

As explained before, large reference value would result in large tracking error of Bu's update law since a logarithmic quantizer is adopted. This is shown in Fig. 3, where three references are considered, i.e. $y_d(t)$, $3y_d(t)$ and $5y_d(t)$. Obviously, as the scale of the reference is enlarged, the maximum tracking error also increases. However, as can be seen from Fig. 4, our algorithm always ensures the tracking error to converge to zero as iteration number increases.

In addition, we compute the quadratic sum of the input errors, i.e. Euclidean norm of δU_k, and show it in Fig. 5. One could find from this plot that the summation of the input error for a whole iteration decreases monotonically along iteration axis.

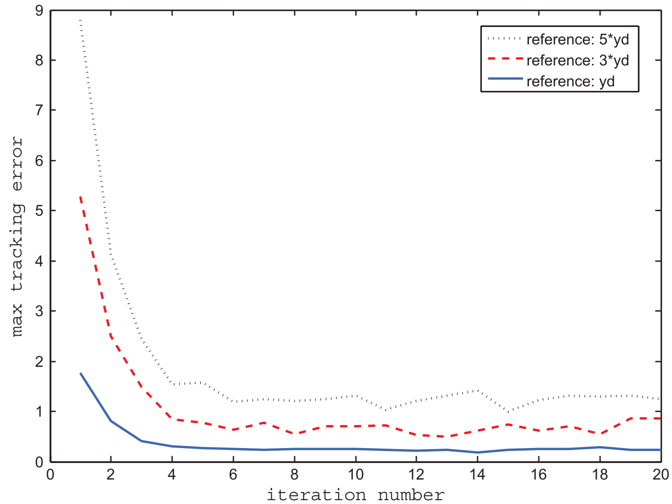

FIG. 3. Maximal tracking error for different references: output quantizer case.

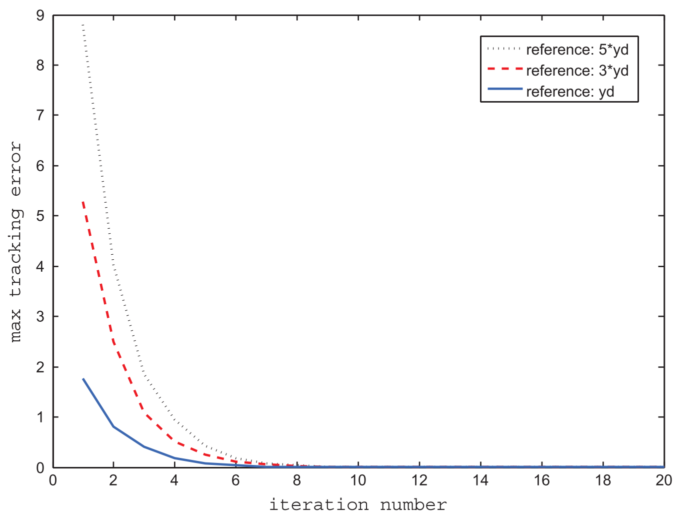

FIG. 4. Maximal tracking error for different references: error quantizer case.

In order to see the effect of quantization density on the tracking performance, we further simulate the example for different scales of density. That is, the density parameter μ is set as 0.95, 0.85, 0.75 and 0.65, respectively. The results are shown in Figs 6 and 7 for the error quantizer case and output quantizer case, respectively. From these figures two observations are given as follows. The first one is that the larger the density is, the better the tracking performance is for both cases. The other one is that different scales of quantization density have little influence in the error quantizer case, while they have great influence in the output quantizer case.

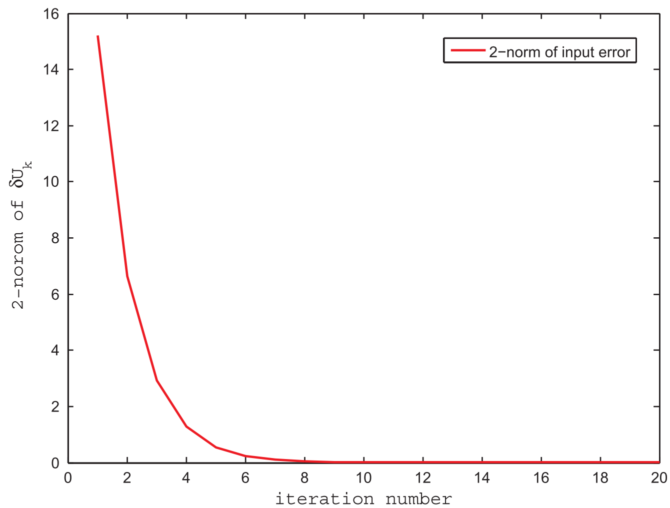

FIG. 5. Quadratic sum of input error.

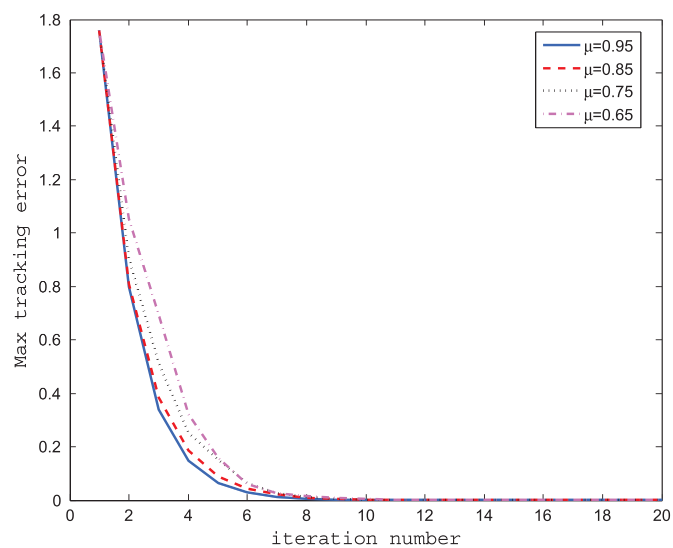

FIG. 6. Maximal tracking error for different scales of density: error quantizer case.

5.2. *Nonlinear system*

Let us consider the following nonlinear system (Bu *et al.*, 2015):

$$\begin{aligned} x_k(t+1) &= -0.75\sin(x_k(t)) + 0.5u_k(t), \\ y_k(t) &= 0.2x_k(t). \end{aligned} \tag{5.2}$$

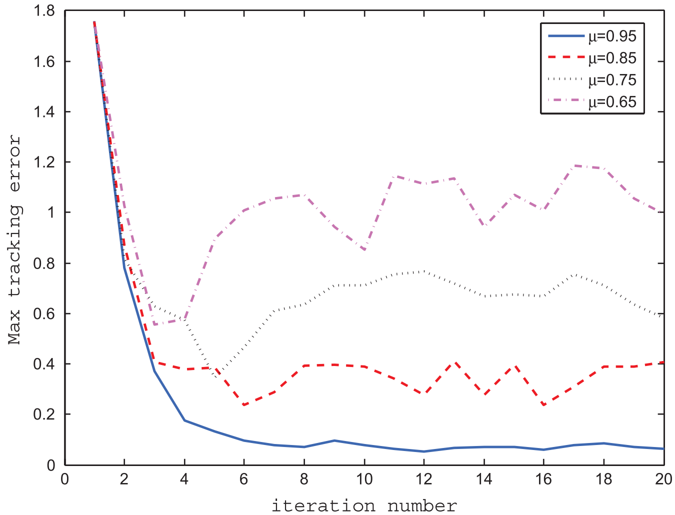

FIG. 7. Maximal tracking error for different scales of density: output quantizer case.

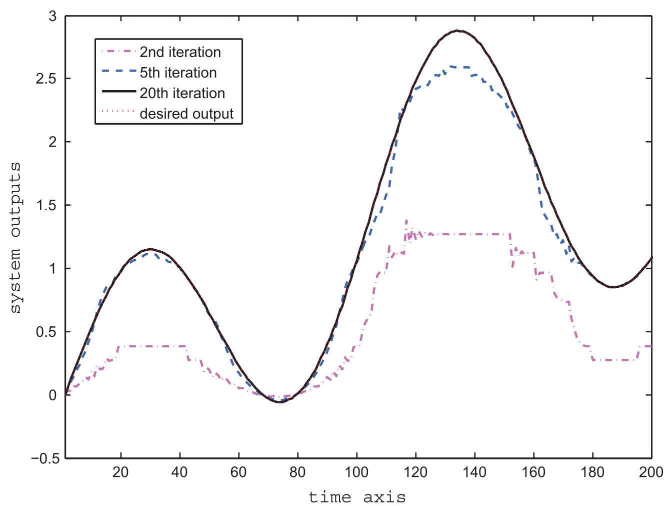

FIG. 8. Outputs at different iterations vs reference.

The desired reference is $y_d(t) = \sin(3t/50) + 1 - \cos(t/50)$, $t \in [0, 200]$. The initial states are set to be $x_k(0) = x_d(0) = 0$ for all k. The initial input is simply chosen as $u_0(t) = 0$, $\forall t$. The parameters in the quantizer are $z_0 = 5$, $\mu = 0.9$, then $\zeta = 0.05$. The learning gain is selected as $L = 5$, then it leads to $\|I - LCB\| + \zeta\|LCB\| = 0.525 < 1$. The algorithm is also performed for 20 iterations.

The tracking performances at the 2nd, 5th and 20th iterations are shown in Fig. 8, where the output at the 20th iteration is also satisfactory.

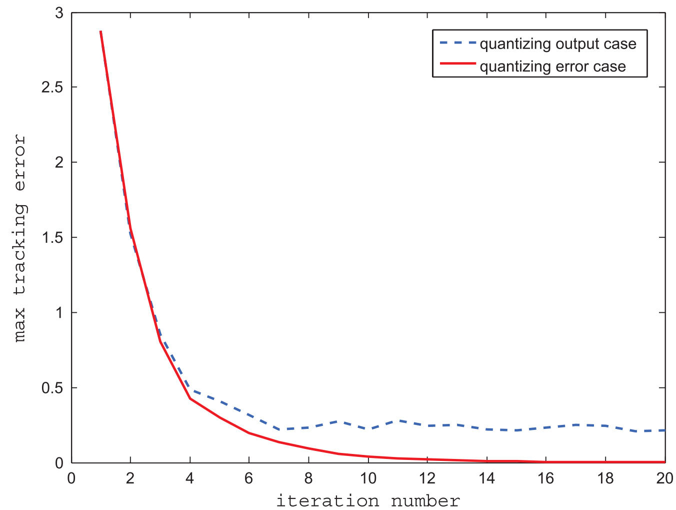

FIG. 9. Maximal tracking error along iteration axis.

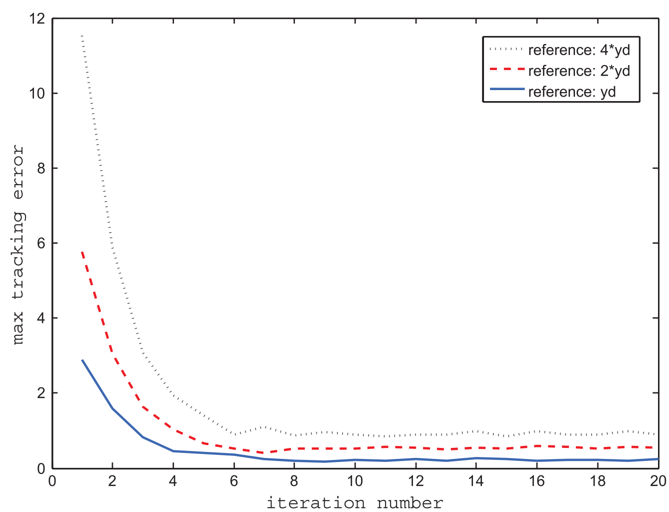

FIG. 10. Maximal tracking error for different references: output quantizer case.

A comparison of maximum tracking error along iteration axis is given in Fig. 9, where solid and dashed lines denote error quantizer and output quantizer cases, respectively. It is evident that using an error quantizer enables us to make the tracking error converge to zero.

Similar to linear system case, maximum errors for different scales of references are provided in Figs 10 and 11 for the output quantizer case and error quantizer case, respectively. The three references used in this example are $y_d(t)$, $2y_d(t)$ and $4y_d(t)$, respectively. From Fig. 10, a non-zero lower bound would always exist for the output quantizer case and the tracking performance reduces as the reference scale increases. However, a zero-error convergence is always guaranteed by the algorithm of this paper,

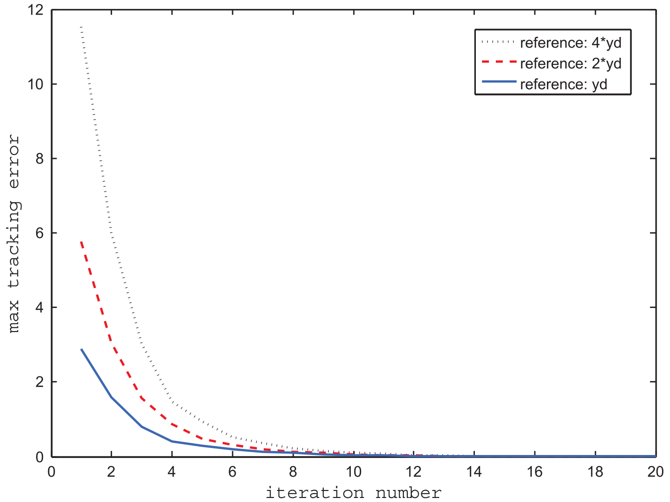

FIG. 11. Maximal tracking error for different references: error quantizer case.

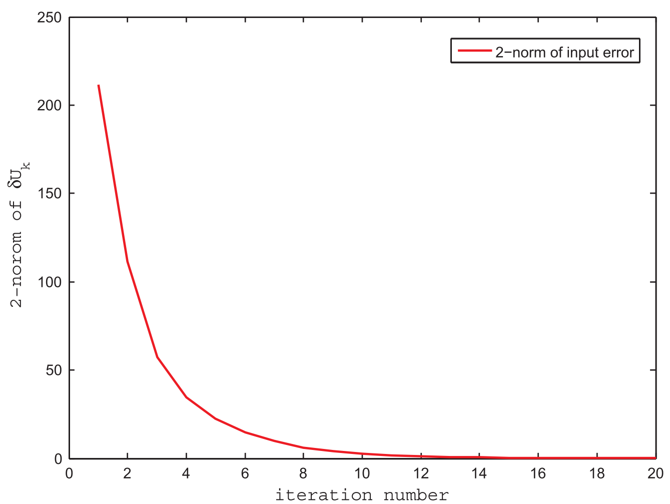

FIG. 12. Quadratic sum of input error.

as revealed in Fig. 11. In addition, Fig. 12 shows the monotonic decreasing trend of quadratic summation of the input error along iteration axis.

Similar to the linear system case, we also simulate for different scales of quantization density and show the results in Figs 13 and 14 for the error quantizer case and output quantizer case, respectively. The observations are also similar to the linear system case.

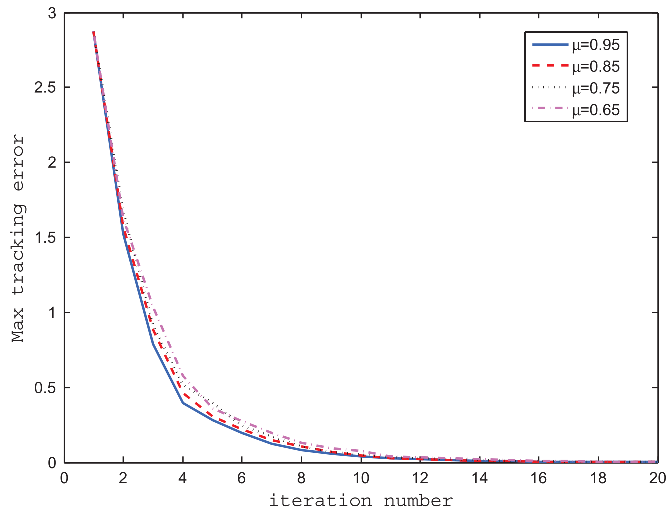

FIG. 13. Maximal tracking error for different scales of density: error quantizer case.

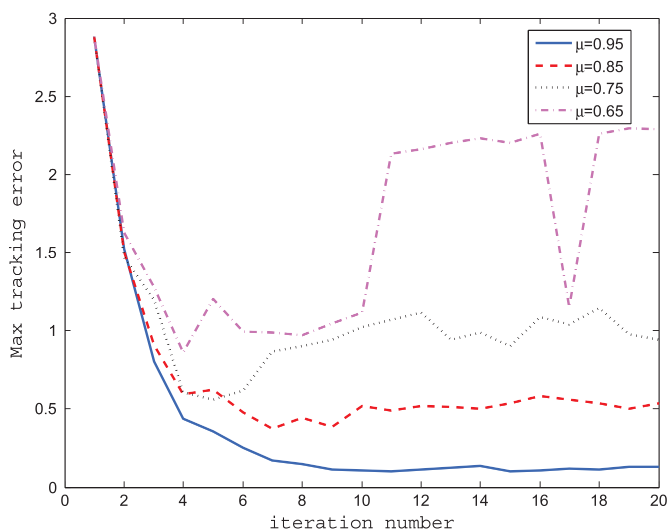

FIG. 14. Maximal tracking error for different scales of density: output quantizer case.

6. Concluding remarks

In this paper, the ILC problem for discrete-time linear and nonlinear systems is discussed by using quantized information. The reference is first transmitted to the plant and then compared with the actual output to derive tracking error. The tracking error is quantized by a logarithmic quantizer and then transmitted back to the controller, which ensures an adaptive learning for precise tracking performance. Using conventional contraction techniques, the tracking error is shown strictly convergent to zero as iteration number goes to infinity. For further research, quantization at the input signal is of great interest.

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References

Ahn, H. S., Chen, Y. Q. & Moore, K. L. (2007) Iterative learning control: survey and categorization from 1998 to 2004. *IEEE Trans. Syst., Man, Cybern. C*, **37**, 1099–1121.

Arimoto, S., Kawamura, S. & Miyazaki, F. (1984) Bettering operation of robots by learning. *J. Robot. Syst.*, **1**, 123–140.

Bristow, D. A., Tharayil, M. & Alleyne, A. G. (2006) A survey of iterative learning control: a learning-based method for high-performance tracking control. *IEEE Control Syst. Mag.*, **26**, 96-114.

Brockett, R. W. & Liberzon, D. (2000) Quantized feedback stabilization of linear systems. *IEEE Trans. Autom. Control*, **45**, 1279–1289.

Bu, X., Hou, Z., Yu, F. & Fu, Z. (2013) Iterative learning control for a class of non-linear switched systems, *IET Control Theory Appl.*, **7**, 470–481.

Bu, X., Hou, Z., Yu, F. & Wang, F. (2014) H-∞ iterative learning controller design for a class of discrete-time systems with data dropouts. *Int. J. Syst. Sci.*, **45**, 1902–1912.

Bu, X., Wang, T., Hou, Z. & Chi, R. (2015) Iterative learning control for discrete-time systems with quantised measurements. *IET Control Theory Appl.*, **9**, 1455–1460.

Curry, R. E. (1970) *Estimation and Control with Quantized Measurements*. Cambridge, MA: MIT Press.

Elia, N. & Mitter, K. (2001) Stabilization of linear systems with limited information. *IEEE Trans. Autom. Control*, **46**, 1384–1400.

Fagnani, F. & Zampieri, S. (2004) Quantized stabilization of linear systems: complexity versus performance. *IEEE Trans. Autom. Control*, **49**, 1534–1548.

Fu, M. & Xie, L. (2005) The sector bound approach to quantized feedback control. *IEEE Trans. Autom. Control*, **50**, 1698–1710.

Jiang, Z.-P. & Liu, T.-F. (2013) Quantized nonlinear control—a survey. *Acta Autom. Sin.*, **39**, 1820–1830.

Liu, N. & Alleyne, A. G. (2014) Iterative learning identification applied to automated off-highway vehicle, *IEEE Trans. Control Syst. Technol.*, **22**, 331–337.

Meng, D., Jia, Y. & Du, J. (2013) Multi-agent iterative learning control with communication topologies dynamically changing in two directions. *IET Control Theory Appl.*, **7**, 261–270.

Meng, D., Jia, Y. & Du, J. (2015) Robust consensus tracking control for multiagent systems with initial state shifts, disturbances, and switching topologies. *IEEE Trans. Neural Netw. Learn. Syst.*, **26**, 809–824.

Meng, D., Jia, Y., Du, J. & Zhang, J. (2014a) High-precision formation control of nonlinear multi-agent systems with switching topologies: a learning approach. *Int. J. Robust Nonlinear Control*, **25**, 1993–2018.

Meng, D., Jia, Y., Du, J. & Zhang, J. (2014b) On iterative learning algorithms for the formation control of nonlinear multi-agent systems. *Automatica*, **50**, 291–295.

Meng, D. & Moore, K. L. (2016) Learning to cooperate: networks of formation agents with switching topologies. *Automatica*, **64**: 278–293.

Shen, D. & Wang, Y. (2014) Survey on stochastic iterative learning control. *J. Process Control*, **24**, 64–77.

Shen, D. & Wang, Y. (2015a) Iterative learning control for networked stochastic systems with random packet losses. *Int. J. Control*, **88**, 959–968.

Shen, D. & Wang, Y. (2015b) ILC for networked nonlinear systems with unknown control direction through random lossy channel. *Syst. Control Lett.*, **77**, 30–39.

Tutty, O., Blackwell, M., Rogers, E. & Sandberg, R. (2014) Iterative learning control for improved aerodynamic load performance of wind turbines with smart rotors. *IEEE Trans. Control Syst. Technol.*, **22**, 967–979.

WANG, L. Y., YIN, G., ZHANG, J.-F. & ZHAO, Y. (2010) *System Identification with Quantized Observations, Theory and Applications*. Boston: Birkhauser.

ZHAO, Y. M., LIN, Y., XI, F. & GUO, S. (2015) Calibration-based iterative learning control for path tracking of industrial robots. *IEEE Trans. Ind. Electron.*, **62**, 2921–2929.