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Brief paper

Distributed learning consensus for heterogenous high-order nonlinear multi-agent systems with output constraints☆

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ABSTRACT

This paper considers the learning consensus problem for heterogenous high-order nonlinear multi-agent systems with output constraints. The dynamics consisting of parameterized and lumped uncertainties is different among different agents. To solve the consensus problem under output constraints, two distributed control protocols are designed with the help of a novel barrier Lyapunov function, which drives the control updating and parameters learning. Both convergence analysis and constraint satisfaction are strictly proved by the barrier composite energy function approach. Illustrative simulations are provided to verify the effectiveness of the proposed protocols.

1. Introduction

In the past decades, multi-agent system (MAS) coordination and control problems have attracted much attention from the control community. Much progress has emerged in formation control, synchronization, flocking, swarm tracking, and containment control among others. For these problems, the consensus framework is an effective approach (Cao, Yu, Ren, & Chen, 2013). The setting of a consensus problem involves triple components, namely, agent model, information exchange topology, and distributed consensus algorithm, respectively. For the agent model, the existing results cover single integrator model (Olfati-Saber & Murray, 2004; Ren, Beard, & Atkins, 2007), double integrator model (Hong, Hu, & Gao, 2006; Ren, 2008; Zhang & Tian, 2009), high-order integrator model (Cui & Jia, 2012), linear system (Scardovi & Sepulchre, 2009; Yu & Wang, 2014), and nonlinear system (Chen & Lewis, 2011; Mehrabian & Khorasani, 2016; Mei, Ren, & Ma, 2011). Moreover, the information exchange topology, described by a graph, has been thoroughly developed in the existing literature (Fang & Antsaklis, 2006; Tahbaz-Salehi & Jadbabaie, 2008). Last, the consensus algorithm is important to generate complex group-level behaviors using simple local coordination rules, which are highly related to practical problems (Khoo, Xie, & Man, 2009; Ren & Beard, 2008; Yang, Tan, & Xu, 2013).

Iterative learning control (ILC) is a matured intelligent control technique to achieve high precision tracking performance by the inherent repetition mechanism (Ahn, Chen, & Moore, 2007; Shen & Wang, 2014; Xu, 2011). Therefore, the ILC strategy has been applied for MASs to achieve learning consensus recently. Ahn and Chen (2009) proposed the first result on formation control using the learning strategy. Later, the reports on satellite trajectory-keeping (Ahn, Moore, & Chen, 2010), mobile robots formation (Chen & Jia, 2010), and coordinated train trajectory tracking (Sun, Hou, & Li, 2013) illustrate successful applications of ILC to MASs. For theoretical research, Yang, Xu, Huang, and Tan (2014, 2015) employed the contraction mapping method for convergence analysis of affine nonlinear MASs. The 2D system technique was used to prove the consensus performance in Meng, Jia, and Du (2013, 2015, 2016) and Meng and Moore (2016) for linear systems. The Lyapunov function method was introduced in Li and Li (2013, 2015, 2016) for MASs where agents were of first-order, second-order and high-order models, respectively. Yang and Xu (2016) also provided a composite energy function (CEF) based analysis for networked Lagrangian systems. While various techniques have been developed for the ILC-based MAS consensus, the existing literature mainly focuses on the conventional system setting without any constraint on the system output.

However, when concerning MASs in the real world, it is found that nearly almost all real systems are subject to certain

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constraints. The constraints arise for the output due to various practical limitations and safety considerations. If we ignore such constraints and conduct the conventional control strategy, the system output may be beyond the tolerant range and lead to serious problems. For example, a platoon of auto-vehicles is a typical MAS, in which the vehicles are required to stay in a regulated range and run within the speed limit all the time. Consequently, when updating the control signal, we should always take these constraints into consideration in order to guarantee a safe drive. Otherwise, traffic accidents would arise for the automatic drive if the vehicle is either out of the road range or over the speed limit. Moreover, due to the physical limitation of wireless networks, there usually exists an upper bound of communication bandwidth in MASs; therefore, the output of each agent should fall in a specified range so that the transmitted data would not exceed the maximal bandwidth. In addition, in consideration of implementation cost, simple and cheap measurement devices are widely used in industrial and automation systems, which may only provide a limited measurement range. In such case, the agent output is required not to exceed the range; otherwise, the output is difficult to measure and then the update cannot proceed. From these observations, we note that the output of each agent in a MAS generally has to satisfy certain constraints, which has not been considered in the existing literature. Once the output constraints are required, it is a natural question how to design and analyze the learning update laws for MASs. This problem motivates the research of this paper. In this paper, we try to propose distributed learning protocols to achieve asymptotical consensus along the iteration axis and guarantee the output constraints simultaneously.

To this end, we apply the idea of barrier Lyapunov function (BLF) similar to Jin and Xu (2013) and Xu and Jin (2013) to handle the output constraints problem. Differing from Jin and Xu (2013) and Xu and Jin (2013), we introduce a general type of BLF and apply it to the design of distributed learning protocols for heterogenous high-order nonlinear MASs. In particular, for a MAS where the dynamics of each agent consists of parameterized and lumped uncertainties, we first define a group of auxiliary functions based on the newly introduced BLF and then apply these functions in the design of the protocols. In this paper, two control protocols are designed. The first one introduces sign functions of the involved quantities to regulate control compensation so that the zero-error asymptotical consensus is achieved while satisfying output constraints. However, such protocol may cause chattering problem due to the frequent sign switching. To facilitate practical applications, we further propose the second control protocol, where the sign function is approximated by a hyperbolic tangent function. In such case, we only guarantee the bounded convergence performance; however, we present a precise estimation of the upper bound, which can help to tune the protocol parameters for a specified consensus performance. We note that Li and Li (2013, 2015, 2016) also applied the CEF method for learning consensus problem. Our paper differs from Li and Li (2013, 2015, 2016) in three aspects: (1) we concentrate on the consensus under output constraints and introduce a general BLF; (2) we provide practical alternative of the algorithm implementations; and (3) we employ distinct analysis techniques.

The rest of the paper is arranged as follows. Section 2 proposes the problem formulation and the general barrier Lyapunov function. Section 3 presents two control protocols and the main theorems, whose proofs are put in the Appendix. Section 4 gives illustrative simulations on an engineering system. Section 5 concludes this paper.

Notations: $\mathscr{G} = (\mathscr{V}, \mathscr{E})$ is a weighted graph. $\mathscr{V} = \{v_1, \ldots, v_N\}$ is a nonempty set of nodes/agents, where N is the number of nodes/agents. $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ is the set of edges/arcs. $(v_i, v_j) \in \mathscr{E}$ indicates that agent j can get information from agent i. $\mathscr{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ denotes the topology of a weighted graph $\mathscr{G}$. a_{ij} is the weighted value, and $a_{ij} = 1$ if $(v_i, v_j) \in \mathscr{E}$, otherwise $a_{ij} = 0$. In addition, $a_{ii} = 0$, $1 \le i \le N$. $d_i = \sum_{j=1}^{N} a_{ij}$ is the in-degree of agent i. $\mathscr{D} = \mathrm{diag}\{d_1, \ldots, d_N\}$ is the in-degree matrix. $\mathscr{L} = \mathscr{D} - \mathscr{A}$ is the Laplacian matrix of a graph $\mathscr{G}$. $\mathscr{N}_i$ denote the set of all neighborhoods of ith agent, where an agent v_j is said to be a neighborhood of agent v_i if v_i can get information from v_j. An agent does not belong to its neighborhood. ε_j denote the access of jth agent to the desired trajectory, that is, $\varepsilon_j = 1$ if agent v_j has direct access to the full information of desired trajectory, otherwise $\varepsilon_j = 0$. $\|\mathbf{x}\|$ denotes the Euclidean norm for a vector $\mathbf{x}$.

2. Problem formulation

Consider a heterogeneous MAS formulated by N ($N > 2$) agents, where the jth agent is modeled by the following high-order nonlinear system

$$\begin{aligned}
\dot{x}_{i,j,k} &= x_{i+1,j,k}, \quad i = 1, \ldots, n-1, \\
\dot{x}_{n,j,k} &= \theta_j^T(t)\xi_{j,k}(t) + b_{j,k}(t)u_{j,k} + \eta_{j,k}(t), \qquad (1)\\
y_{j,k} &= \{x_{1,j,k}, x_{2,j,k}\},
\end{aligned}$$

where $i = 1, 2, \ldots, n$ denotes the ith dimension of state, $j = 1, 2, \ldots, N$ denotes the agent, and $k = 1, 2, \ldots$ is the iteration number. Denote the state of the jth agent at the kth iteration as $x_{j,k} \triangleq [x_{1,j,k}, \ldots, x_{n,j,k}]^T$. $\theta_j^T(t)\xi_{j,k}(t)$ is the parametric uncertainty, where $\theta_j(t)$ is an unknown parameter vector of the jth agent, which is continuous and bounded on the operation interval $[0, T]$, while $\xi_{j,k}(t) \triangleq \xi_j(x_{j,k}, t)$ is a known time-varying vector-function. $b_{j,k}(t) \triangleq b_j(x_{j,k}, t)$ is the unknown time-varying control gain. $\eta_{j,k}(t) \triangleq \eta_j(x_{j,k}, t)$ is the unknown lumped uncertainty with a known upper bounded function $|\eta_j(x_{j,k}, t)| \le \rho(x_{j,k}, t)$. In the following, denote $\xi_{j,k} \triangleq \xi_j(x_{j,k}, t)$, $b_{j,k} \triangleq b_j(x_{j,k}, t)$, $\eta_{j,k} \triangleq \eta_j(x_{j,k}, t)$, and $\rho_{j,k} \triangleq \rho(x_{j,k}, t)$ where no confusion arises. The system output $y_{j,k} = \{x_{1,j,k}, x_{2,j,k}\}$ can be either $x_{1,j,k}$ or $x_{2,j,k}$ or both, but cannot be varying. For the high-order system, it is required that the outputs should satisfy the given boundedness constraints.

Remark 1. The agent model (1) was also investigated in Li and Li (2016), where the input gain is set to be one and the lumped uncertainty is bounded by a constant. The model (1) for a single system was also considered in Jin and Xu (2013), where the lumped uncertainty is assumed to be variation-norm-bounded. In such case, the tracking reference is assumed to take the same structure of the system model. In this paper, all these requirements are removed. In addition, the model (1) represents a wide range of system uncertainties, as the neural networks and fuzzy approximation-based transformations of general nonlinear systems usually conform to this model.

Let the desired trajectory (virtual leader) be x_r, $x_r \triangleq [x_{1,r}, \ldots, x_{n,r}]^T$ satisfying that $\dot{x}_{i,r} = x_{i+1,r}$, $1 \le i \le n-1$ and $\dot{x}_{n,r} = f(t, x_r)$ with bounded $f(t, x_r)$.

The following assumptions are required for analysis.

A1 Assume that the input gain $b_{j,k}$ does not change its sign. Meanwhile, it has lower and upper bounds. That is, we assume $0 < b_{\min} \le b_{j,k} \le b_{\max}$, where $b_{\min}$ is known.

A2 Each agent satisfies the alignment condition, $x_{j,k}(0) = x_{j,k-1}(T)$. In addition, the desired trajectory is spatially closed, that is, $x_r(0) = x_r(T)$.

Remark 2. In the conventional ILC literature, the so-called identical initialization condition (i.i.c.), i.e., $x_{j,k}(0) = x_r(0)$ for all agents and iterations, is the most common assumption for iteration re-initialization. However, this condition is difficult to satisfy for many MASs as it requires both time and spatial resetting for all agents. In this paper, we employ the alignment condition, in which the spatial resetting is removed. In other words, we only require that the new iteration starts from the position where they stop in the previous iteration. Such condition is widely satisfied in motion systems and manipulator systems.

Denote the tracking error of the jth agent to the desired trajectory as $e_{j,k} \triangleq x_{j,k} - x_r = [e_{1,j,k}, \dots, e_{n,j,k}]^T$. However, not all agents can access the desired trajectory. Thus, the tracking error $e_{j,k}$ is available only for a part of the agents that the virtual leader is within their neighborhood. Meanwhile, any agent could acknowledge the information of its neighbor agents. Therefore, for the jth agent, we define the extended observation error as $z_{j,k} \triangleq [z_{1,j,k}, \dots, z_{n,j,k}]^T = \sum_{l=1}^{N} a_{jl}(x_{j,k} - x_{l,k}) + \varepsilon_j(x_{j,k} - x_r)$.

The control objective of the heterogenous high-order MAS is to design distributed control protocols such that the tracking error converges to zero and the specified boundedness constraints of outputs are ensured for all agents.

To obtain a compact form of the MAS, denote $\bar{e}_{i,k}$, $\bar{x}_{i,k}$, and $\bar{z}_{i,k}$ as the stack of tracking errors, states, and extended observation errors for all agents at the ith dimension, i.e., $\bar{e}_{i,k} = [e_{i,1,k}, \dots, e_{i,N,k}]^T$, $\bar{x}_{i,k} = [x_{i,1,k}, \dots, x_{i,N,k}]^T$, $\bar{z}_{i,k} = [z_{i,1,k}, \dots, z_{i,N,k}]^T$. Noting $\mathscr{L}\mathbf{1} = 0$, we have

$$\bar{z}_{i,k} = \mathscr{L}(\bar{x}_{i,k} - \mathbf{1}x_{i,r}) + \mathscr{B}\bar{e}_{i,k} = (\mathscr{L} + \mathscr{B})\bar{e}_{i,k}, \tag{2}$$

where $\mathscr{B} = \text{diag}\{\varepsilon_1, \dots, \varepsilon_N\}$ and $\mathbf{1} = [1, \dots, 1]^T \in \mathbb{R}^N$. Let $\mathscr{H} = \mathscr{L} + \mathscr{B}$.

We give the assumption on communication topology.

A3 The graph is fixed and directed. The virtual leader is globally reachable in the extended graph $\overline{\mathscr{G}}$ consisting of N agents and the virtual leader.

Remark 3. Assumption A3 assumes that the virtual leader is directly accessed to a part of agents and globally reachable for all agents. Here, by globally reachable we mean there is a path from the virtual leader to the agent possibly passing several other agents (denoting the information transmission direction). This assumption is necessary for a leader–follower consensus tracking problem. We note that several papers presented the directed and switching topologies (Meng et al., 2015, 2016; Meng & Moore, 2016). The pivotal principle of convergence in these papers is to ensure a contraction or joint contraction for all possible topologies. Thus, the systems are generally linear and the learning gain matrix depends on the graph information. In this paper, we concentrate on high-order nonlinear systems with output constraints and provide a new BLF for the solution. Our results can be extended to switching graph following the main procedures but with additional requirements and derivations. We restrict our discussions to fixed graph to present a concise proof of the main results.

Based on A3, we can conclude that $\mathscr{H}$ is a positive stable matrix as $\mathscr{B}$ is a nonnegative diagonal matrix (Hu & Hong, 2007; Lin, Francis, & Maggiore, 2005). Let us denote the minimum and maximum singular values as $\sigma_{\min}(\mathscr{H})$ and $\sigma_{\max}(\mathscr{H})$.

To ensure output constraints, we introduce a general BLF satisfying the following definition.

Definition 1. We call a BLF $V(t) = V(\gamma^2(t), k_b)$ "γ-type BLF" if all the following conditions hold.

- $V \to \infty$ if and only if $\gamma^2 \to k_b^2$, where k_b is a certain fixed parameter in V, provided that $\gamma^2(0) < k_b^2$.
- $V \to \infty$ if and only if $\frac{\partial V}{\partial \gamma^2} \to \infty$.
- If $\gamma^2 < k_b^2$, then $\frac{\partial V}{\partial \gamma^2} \geq C$, where $C > 0$ is a constant.
- $\lim_{k_b \to \infty} V(\gamma^2(t), k_b) = \frac{1}{2}\gamma^2(t)$.

Remark 4. The first item is to ensure the boundedness of γ^2 as long as the BLF is finite, so it is fundamental. The second item is to show the boundedness of the BLF by making use of $\frac{\partial V}{\partial \gamma^2}$ in the controller design. The third item offers a flexibility of the BLF as can be seen in the proofs of our main theorems. From the last item, the newly defined γ-type BLF can be regarded as a general form of the conventional quadratic Lyapunov function, in the sense that they are mathematically equivalent when $k_b \to \infty$. Two typical examples are found in the literature: the log-type, $V(t) = \frac{k_b^2}{2}\log\left(\frac{k_b^2}{k_b^2 - \gamma^2(t)}\right)$, and the tan-type, $V(t) = \frac{k_b^2}{\pi}\tan\left(\frac{\pi\gamma^2(t)}{2k_b^2}\right)$. By direct calculations, one can find that all the items of the definition are satisfied.

In the following, to simplify notations, the time and state dependence of the system may be omitted whenever no confusion arises.

3. Main results

In order to make the analysis clear to follow, we first introduce auxiliary functions for the use of backstepping techniques. The fictitious errors are defined as follows

$$\begin{aligned} \gamma_{1,j,k} &= z_{1,j,k} = \varepsilon_j(x_{1,j,k} - x_{1,r}) + \sum_{l=1}^{N} a_{jl}(x_{1,j,k} - x_{1,l,k}), \\ \gamma_{i,j,k} &= (\varepsilon_j + d_j)x_{i,j,k} - \sigma_{i-1,j,k}, \quad i = 2, \dots, n, \end{aligned} \tag{3}$$

where the stabilizing functions $\sigma_{i,j,k}$ are defined as

$$\begin{aligned} \sigma_{1,j,k} &= (\varepsilon_j \dot{x}_{1,r} + \sum_{l=1}^{N} a_{j,l}\dot{x}_{1,l,k}) - \lambda_{1,j,k}^{-1}\mu_{1,j}\gamma_{1,j,k}, \\ \sigma_{i,j,k} &= \dot{\sigma}_{i-1,j,k} - \lambda_{i,j,k}^{-1}\mu_{i,j}\gamma_{i,j,k} - \lambda_{i,j,k}^{-1}\lambda_{i-1,j,k}\gamma_{i-1,j,k}, \end{aligned} \tag{4}$$

for $i = 2, \dots, n$, and

$$\lambda_{i,j,k} = \lambda_{i,j,k}(t) = \frac{1}{\gamma_{i,j,k}}\frac{\partial V_{i,j,k}}{\partial \gamma_{i,j,k}}, \quad V_{i,j,k} = V(\gamma_{i,j,k}^2, k_{b_{i,j}}). \tag{5}$$

Here $V(\cdot)$ is the γ-type BLF. $k_{b_{1,j}} > 0$ and $k_{b_{2,j}} > 0$ are the constraints for $\gamma_{1,j,k}$ and $\gamma_{2,j,k}$ of the jth agent, $\forall k$, and $k_{b_{i,j}} > 0$, $i = 3, \dots, n$ are virtual bounds on $\gamma_{i,j,k}$ that can be taken arbitrarily large values, $\forall k$. $\mu_{i,j}$ is a positive constant to be designed later.

Based on the above notations, we can now propose the control protocols for the MAS to achieve uniform state tracking consensus and prevent output constraints violation.

$$\begin{aligned} u_{j,k} =& \hat{u}_{j,k} - \frac{1}{b_{\min}}\hat{\theta}_{j,k}^T\xi_{j,k}\text{sgn}\left(\lambda_{n,j,k}\gamma_{n,j,k}\hat{\theta}_{j,k}^T\xi_{j,k}\right) \\ &- \frac{1}{b_{\min}(\varepsilon_j + d_j)}\sigma_{n,j,k}\text{sgn}\left(\lambda_{n,j,k}\gamma_{n,j,k}\sigma_{n,j,k}\right) \\ &- \frac{1}{b_{\min}(\varepsilon_j + d_j)}\rho_{j,k}\text{sgn}\left(\rho_{j,k}\lambda_{n,j,k}\gamma_{n,j,k}\right) \end{aligned} \tag{6}$$

with iterative updating laws

$$\hat{u}_{j,k} = \hat{u}_{j,k-1} - q_j\lambda_{n,j,k}\gamma_{n,j,k}, \tag{7}$$

$$\hat{\theta}_{j,k} = \hat{\theta}_{j,k-1} + p_j\lambda_{n,j,k}\gamma_{n,j,k}\xi_{j,k}, \tag{8}$$

where $q_j > 0$ and $p_j > 0$ are design parameters, $\forall j = 1, \dots, N$. sgn$(\cdot)$ is a sign function; that is, sgn(χ) is equal to $+1$ for $\chi > 0$, 0 for $\chi = 0$, and -1 for $\chi < 0$, respectively. The initial values of the iterative update laws are set to be zero, i.e., $\hat{u}_{j,0} = 0$, $\hat{\theta}_{j,0} = 0$, $\forall j = 1, \dots, N$.

We have the following consensus theorem, whose proof is given in the Appendix.

Theorem 1. *Assume that A1–A3 hold for the multi-agent system (1). The closed-loop system consisting of model (1) and control algorithms (6)–(8), can ensure that:*

(i) the tracking error $e_{j,k}(t)$ converges to zero uniformly as the iteration number k goes to infinity, $\forall j = 1, \ldots, N$;

(ii) the system output, which is $x_{1,j,k}$ or $x_{2,j,k}$ or both, is bounded by predefined constraints; that is, $|x_{1,j,k}| < k_{s,1}$ and $|x_{2,j,k}| < k_{s,2}$ are guaranteed for all iterations and agents.

Remark 5. If the lumped uncertainty is norm-bounded with an unknown coefficient ω: $|\eta_{j,k}| \leq \omega\rho(x_{j,k}, t)$, then an additional estimation process could be established for this coefficient and the robust compensation term is appended to the controller based on the newly estimated parameter similarly to the parameterized uncertainty part.

Generally speaking, the sign function used in the algorithm (6) makes itself possibly discontinuous, which may lead to the problem of existence and uniqueness of solutions. Moreover, it may also cause chattering that might excite high-frequency unmodeled dynamics. This motivates us to seek an appropriate smooth approximation of the sign function for practical applications. In the following, we take a hyperbolic tangent function as an alternative. A lemma demonstrating the compensation property of the hyperbolic tangent function is given as follows.

Lemma 1 (*Polycarpous & Ioannouq, 1996*)**.** *For any $\varepsilon > 0$ and for any $\chi \in \mathbb{R}$, we have $0 \leq |\chi| - \chi \tanh\left(\frac{\chi}{\varepsilon}\right) \leq \delta\varepsilon$, where δ is a constant that satisfies $\delta = e^{-(\delta+1)}$, i.e., $\delta = 0.2785$.*

Now the algorithm (6) becomes the following one

$$\begin{aligned} u_{j,k} =& \hat{u}_{j,k} - \frac{1}{b_{\min}}\hat{\theta}_{j,k}^T\xi_{j,k}\tanh\left(\frac{\lambda_{n,j,k}\gamma_{n,j,k}\hat{\theta}_{j,k}^T\xi_{j,k}}{\varepsilon}\right) \\ &- \frac{1}{b_{\min}(\varepsilon_j + d_j)}\sigma_{n,j,k}\tanh\left(\frac{\lambda_{n,j,k}\gamma_{n,j,k}\sigma_{n,j,k}}{\varepsilon}\right) \\ &- \frac{1}{b_{\min}(\varepsilon_j + d_j)}\rho_{j,k}\tanh\left(\frac{\rho_{j,k}\lambda_{n,j,k}\gamma_{n,j,k}}{\varepsilon}\right). \end{aligned} \tag{9}$$

From Lemma 1, a constant compensation error always exists in the difference and differential expressions of $V_{j,k}^1(t)$. Hence it is impossible to derive that the difference of $E_k(T)$ will be negative even after sufficiently many iterations. Consequently, only a bounded convergence can be obtained in the following theorem and its proof is put in the Appendix.

Theorem 2. *Assume that A1– A3 hold for the multi-agent system (1). The closed-loop system consisting of model (1) and control algorithms (7)– (9) can ensure that the summation of L_T^2-norm of fictitious errors $\sum_{j=1}^N\sum_{i=1}^n \int_0^T \gamma_{i,j,k}^2 d\tau$ converges to the ζ-neighborhood of zero within finite iterations, where $\zeta = 3TN\bar{\delta}\varepsilon/\mu_m + \nu$ with T, N, $\bar{\delta}$ being the iteration length, amount of agents, a constant satisfying $\bar{\delta} > \frac{b_{j,k}}{b_{\min}}(\varepsilon_j + d_j)\delta$, $\forall j$, and $\nu > 0$ being an arbitrary small constant. Then, the summation of L_T^2-norm of all tracking errors $\sum_{j=1}^N\sum_{i=1}^n \int_0^T e_{i,j,k}^2 d\tau$ converges to the ζ_e-neighborhood of zero within finite iterations, where*

$$\zeta_e \triangleq \frac{3\kappa^2 NT\bar{\delta}\varepsilon}{\sigma_{\min}^2(\mathscr{H})\mu_m} + \frac{\kappa^2\nu}{\sigma_{\min}^2(\mathscr{H})} \tag{10}$$

with κ being a constant defined later.

From the proof in the Appendix, it is seen that the output constraint verification is difficult to achieve, because the boundedness of $E_k(t)$ is no longer guaranteed technically. To overcome this problem, we replace the updating laws (7)–(8) with the following practical dead-zone updating laws

$$\hat{u}_{j,k} = \begin{cases} \hat{u}_{j,k-1} - q_j\lambda_{j,k}\gamma_{j,k}, & \text{if } \int_0^T \|z_{j,k-1}\|^2 d\tau > \varsigma, \\ \hat{u}_{j,k-1}, & \text{otherwise,} \end{cases} \tag{11}$$

$$\hat{\theta}_{j,k} = \begin{cases} \hat{\theta}_{j,k-1} + p_j\lambda_{j,k}\gamma_{j,k}\xi_{j,k}, & \text{if } \int_0^T \|z_{j,k-1}\|^2 d\tau > \varsigma, \\ \hat{\theta}_{j,k-1}, & \text{otherwise,} \end{cases} \tag{12}$$

where $q_j > 0$, $p_j > 0$, $\forall j = 1, \ldots, N$ are design parameters. The initial values are set to be zero, i.e., $\hat{u}_{j,0} = 0$, $\hat{\theta}_{j,0} = 0$, $\forall j = 1, \ldots, N$. The prior defined parameter ς denotes the bound of the convergent neighborhood.

Remark 6. The essential mechanism of (11)–(12) is that learning processes of $\hat{u}_{j,k}$ and $\hat{\theta}_{j,k}$ will stop updating whenever the extended observation error enters the predefined neighborhood of zero, so that the control system would repeat the same tracking performance since then. Consequently, the boundedness of (11)–(12) and the output constraints condition are fulfilled naturally as long as the bounded convergence is finished within finite iterations. This observation is summarized in the following corollary.

Corollary 1. *Assume that A1–A3 hold for the multi-agent system (1). The closed-loop system consisting of model (1) and control algorithms (9) and (11)–(12) can ensure the following properties.*

(i) The extended observation errors converge to the predefined ς-neighborhood of zero within finite iterations in the sense of L_T^2-norm, i.e., $\int_0^T \|z_{j,k-1}\|^2 d\tau < \varsigma$ within finite iterations, $\forall j$. Consequently, the tracking errors would converge to a corresponding neighborhood of zero in the sense of L_T^2-norm whose upper bound is defined as $\varsigma/\sigma_{\min}^2(\mathscr{H})$.

(ii) Both $\hat{u}_{j,k}$ and $\hat{\theta}_{j,k}$ are bounded in the sense of L_T^2-norm, $\forall j = 1, \ldots, N$, $\forall k$.

(iii) The system output, which is $x_{1,j,k}$ or $x_{2,j,k}$ or both, is bounded by predefined constraint; that is, $|x_{1,j,k}| < k_{s,1}$ and $|x_{2,j,k}| < k_{s,2}$ are guaranteed for all iterations and agents.

The proof is put in the Appendix.

4. Illustrative simulations

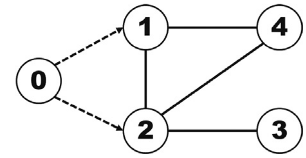

Fig. 1. Communication graph among agents in the network.

To illustrate the applications of the proposed algorithms, consider a group of four agents. The communication topology is demonstrated in Fig. 1, where vertex 0 represents the desired reference or virtual leader and the dashed lines stand for the communication links between leader and followers. In this simulation, agents 1 and 2 can access the information from the leader. The solid lines stand for the communication links among the four agents.

In the simulation, the agent dynamics is modeled by a one-link robotic manipulator (Xu & Xu, 2004):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{ml^2 + I} \end{bmatrix}[u - gl\cos x_1 + \eta_1]$$

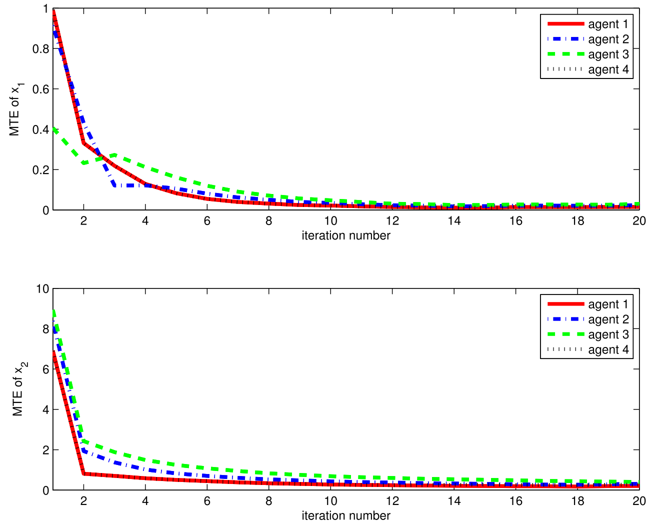

Fig. 2. Maximal tracking error profiles: (upper) the 1st dimension (lower) the 2nd dimension.

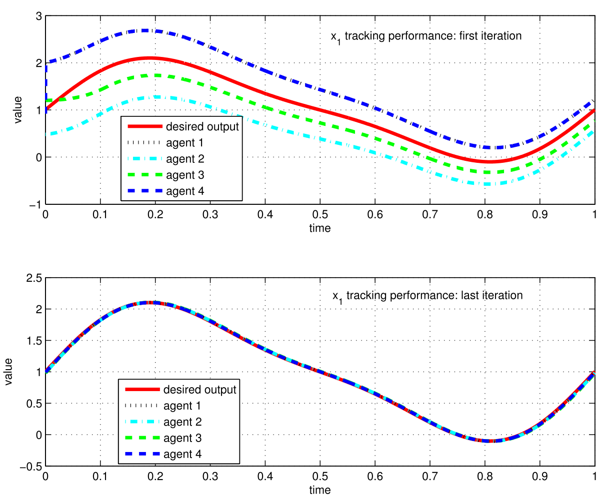

Fig. 3. Tracking profiles of the first dimension: x_1.

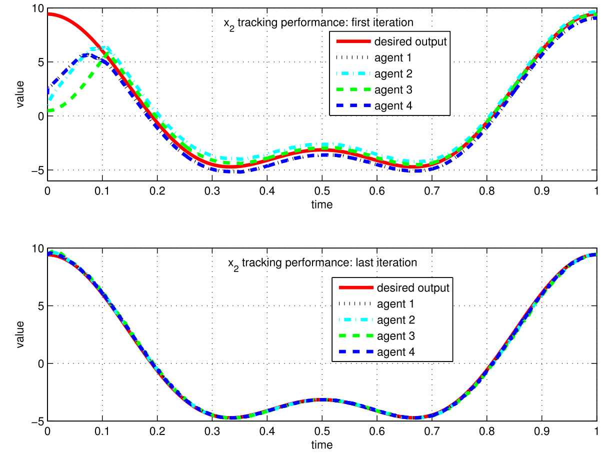

Fig. 4. Tracking profiles of the second dimension: x_2.

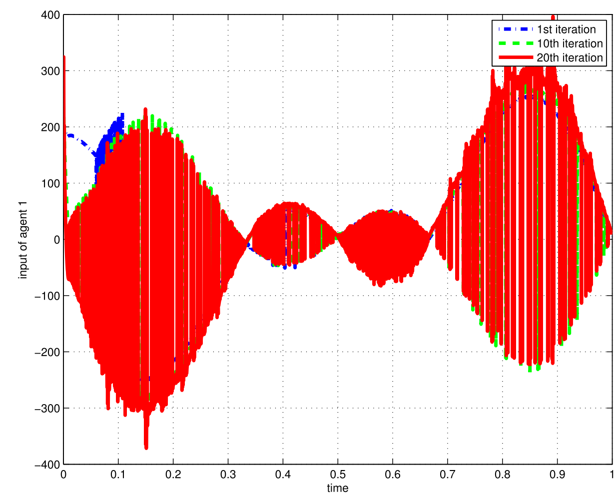

Fig. 5. Input profiles for agent 1 using (6)–(8).

where x_1 is the joint angle, x_2 is the angular velocity, m is the mass, l is the length, I is the moment of inertia, and u is the joint input. $\eta_{j,k}(t) = h_1 \sin(\omega_1 t) + h_2 \sin(\omega_2 t)$ denotes unknown uncertainty, where h_1 and h_2 are random variables subject to uniform distribution in [0, 1], while ω_1 and ω_2 are random variables subject to uniform distribution in [0, 10]. The input gain is $b = 1/(ml^2 + I)$. Let $m = (3 + 0.1 \sin t)$ kg, $l = 1$ m, $I = 0.5$ kg m^2, and g be the gravitational acceleration. In order to simulate the heterogenous MAS, we let $\theta_1 = -gl/(ml^2 + I)$ and $\theta_i = \theta_1 + 0.1(i-1)$, $i = 2, 3, 4$. Clearly, $\xi_{j,k}(t) = \cos(x_{1,j,k})$. The initial states for the first iteration are set to be $\bar{x}_{1,0} = [1.5\ 0.88\ 1.2\ 0.92]^T$, $\bar{x}_{2,0} = 5 \times [0.5\ 0.2\ 0.1\ 0.4]^T$.

The tracking reference is given as $x_{1,r} = 1 + \sin(2\pi t) + 0.25 \sin(4\pi t)$ and $x_{2,r} = 2\pi \cos(2\pi t) + \pi \cos(4\pi t)$, $t \in [0, T]$ with $T = 1$. The BLF chooses the log-type given in Remark 4 with $k_{b_{1,j}} = 10$ and $k_{b_{2,j}} = 10$. The simulations are run for 20 iterations for each control scheme.

We first simulate the original algorithms (6)–(8). The parameters in the algorithms are selected as $b_{\min} = 0.25$, $q_j = 50$, and $p_j = 1$. The parameters in the stabilizing functions are $\mu_{1,j} = 1$ and $\mu_{2,j} = 5$. Define the maximum tracking error (MTE) as $\max_t |x_{i,j,k} - x_{i,r}|$ for the ith dimension of the jth agent at the kth iteration, $i = 1,2$, $j = 1, 2, 3, 4$. The MTE profiles for all agents along the iteration axis are shown in Fig. 2. As one can see, the MTEs for all agents are reduced a lot during the first several iterations.

The trajectories of all agents at the 1st and 20th iterations are shown in Figs. 3 and 4 for x_1 and x_2, respectively. In each figure, the upper subplot shows the case of the 1st iteration, where all trajectories do not match the desired reference, whereas the lower subplot shows the case of the 20th iteration, where all trajectories coincide with the desired reference.

The input profiles of Agent 1 at the 1st, 10th, and 20th iterations are shown in Fig. 5. Clearly, the input file at the 20th iteration has a heavy chattering problem. The input profiles for the other three agents have similar performance.

This observation motivates us to further consider the approximation case. We simulate the smoothed algorithms (9) and (7)–(8). The parameter in the tanh function is $\varepsilon = 10$. The other parameters are the same as the above. The input profiles for the 1st, 10th, and 20th iterations are shown in Fig. 6, where the chattering problem has been overcome. Meanwhile, the tracking performance is similar to the performance shown in Figs. 2–4. Thus, we omit the figures to save space.

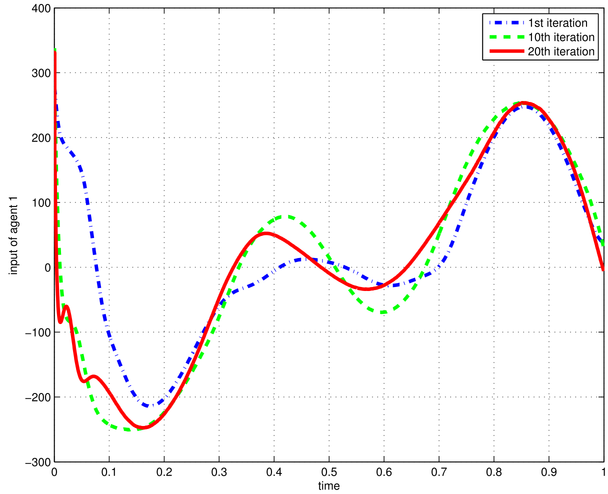

Fig. 6. Input profiles for agent 1 using (9) and (7)–(8).

5. Conclusions

In this paper, we have addressed the distributed learning consensus problem for a heterogenous high-order nonlinear MAS with output constraints. We introduce a novel barrier Lyapunov function to handle the output constraints and propose two consensus protocols. The first control protocol includes sign functions of involved quantities for regulating the uncertainty compensation, and the consensus convergence and constraint satisfaction can be proved by the BCEF approach. However, the sign functions in this protocol may cause chattering. Therefore, we proceed to present the second control protocol, where the sign function is approximated by a hyperbolic tangent function. In such case, the bounded consensus is established with a precise estimation of the upper bound. A practical implementation of the learning processes is also proposed to guarantee the output constraints. For further research, it is of great significance to consider the directed and switching topologies, for which some assumptions in this paper should be revised.

Appendix

Proof of Theorem 1. The proof consists of five parts. First, we investigate the decreasing property of the given BCEF in the iteration domain. By checking the derivative of the BCEF, the finiteness of the BCEF and the boundedness of involved quantities are shown in Part II. Next, we prove the convergence of extended observation errors. In Part IV, the satisfaction of output constraints is verified for all iterations. Last, the uniform consensus tracking is provided.

Define the following BCEF:

$$E_k(t)=\sum_{j=1}^{N}E_{j,k}(t)=\sum_{j=1}^{N}(V_{j,k}^1(t)+V_{j,k}^2(t)+V_{j,k}^3(t)), \tag{13}$$

$$V_{j,k}^1(t)=\sum_{i=1}^{n}V_{i,j,k}(t)=\sum_{i=1}^{n}V(\gamma_{i,j,k}^2(t),t), \tag{14}$$

$$V_{j,k}^2(t)=\frac{(\varepsilon_j+d_j)}{2p_j}\int_0^t(\hat{\theta}_{j,k}-\theta_j)^T(\hat{\theta}_{j,k}-\theta_j)d\tau, \tag{15}$$

$$V_{j,k}^3(t)=\frac{(\varepsilon_j+d_j)}{2q_j}\int_0^t b_{j,k}\hat{u}_{j,k}^2 d\tau. \tag{16}$$

Part I: Difference of $E_k(t)$

First, we show the decreasing property of $E_k(t)$ along the iteration axis at $t=T$. The difference of $E_k(T)$ is defined as $\Delta E_k(T)\triangleq E_k(T)-E_{k-1}(T)=\sum_{j=1}^{N}\Delta E_{j,k}(T)=\sum_{j=1}^{N}(\Delta V_{j,k}^1(T)+\Delta V_{j,k}^2(T)+\Delta V_{j,k}^3(T))$. We will examine the three terms $\sum_{j=1}^{N}\Delta V_{j,k}^1(T)$, $\sum_{j=1}^{N}\Delta V_{j,k}^2(T)$, $\sum_{j=1}^{N}\Delta V_{j,k}^3(T)$ in sequence. Considering $\Delta V_{j,k}^1(T)=\sum_{i=1}^{n}\Delta V_{i,j,k}(T)$, we will show the case for $i=1,2$ and generalize it for all i. Starting from $i=1$, we have

$$\Delta V_{1,j,k}(T)=V(\gamma_{1,j,k}^2(0))-V(\gamma_{i,j,k-1}^2(T))+\int_0^T\frac{1}{\gamma_{1,j,k}}\frac{\partial V_{1,j,k}}{\partial\gamma_{1,j,k}}\gamma_{1,j,k}\dot{\gamma}_{1,j,k}d\tau.$$

Note that $\gamma_{1,j,k}(0)=z_{1,j,k}(0)=\sum_{l=1}^{N}a_{jl}(x_{1,j,k}(0)-x_{1,l,k}(0))+\varepsilon_j(x_{1,j,k}(0)-x_{1,r}(0))=\sum_{l=1}^{N}a_{jl}(x_{1,j,k-1}(T)-x_{1,l,k-1}(T))+\varepsilon_j(x_{1,j,k-1}(T)-x_{1,r}(T))=z_{1,j,k-1}(T)=\gamma_{1,j,k-1}(T)$. Thus, $\Delta V_{1,j,k}=\int_0^T\lambda_{1,j,k}\gamma_{1,j,k}\dot{\gamma}_{1,j,k}d\tau$.

By the definition of $\gamma_{1,j,k}$, we have $\dot{\gamma}_{1,j,k}=\dot{z}_{1,j,k}=(\varepsilon_j+d_j)\dot{x}_{1,j,k}-(\varepsilon_j\dot{x}_{1,r}+\sum_{l\in\mathcal{N}_j}\dot{x}_{1,l,k})=(\varepsilon_j+d_j)x_{2,j,k}-\sigma_{1,j,k}-\lambda_{1,j,k}^{-1}\mu_{1,j}\gamma_{1,j,k}=\gamma_{2,j,k}-\lambda_{1,j,k}^{-1}\mu_{1,j}\gamma_{1,j,k}$. Thus, $\int_0^T\lambda_{1,j,k}\gamma_{1,j,k}\dot{\gamma}_{1,j,k}d\tau=\int_0^T\left(\lambda_{1,j,k}\gamma_{1,j,k}\gamma_{2,j,k}-\mu_{1,j}\gamma_{1,j,k}^2\right)d\tau$.

Next we proceed to $\Delta V_{2,j,k}(T)$, which is

$$\Delta V_{2,j,k}(T)=V(\gamma_{2,j,k}^2(0))-V(\gamma_{2,j,k-1}^2(T))+\int_0^T\frac{1}{\gamma_{2,j,k}}\frac{\partial V_{2,j,k}}{\partial\gamma_{2,j,k}}\gamma_{2,j,k}\dot{\gamma}_{2,j,k}d\tau.$$

Since $\gamma_{2,j,k}=(\varepsilon_j+d_j)x_{2,j,k}-\sigma_{1,j,k}=(\varepsilon_j+d_j)x_{2,j,k}-(\varepsilon_j x_{2,r}+\sum_{l=1}^{N}a_{jl}x_{2,l,k})+\lambda_{1,j,k}^{-1}\mu_{1,j}\gamma_{1,j,k}$, $x_{2,j,k}(0)=x_{2,j,k}(T)$, $\forall j=1,\dots,N$, $x_{2,r}(0)=x_{2,r}(T)$, $\gamma_{1,j,k}(0)=\gamma_{1,j,k}(T)$, and $\lambda_{1,j,k}^{-1}$ is a function of $\gamma_{1,j,k}$, we have $\gamma_{2,j,k}(0)=\gamma_{2,j,k}(T)$. Then, it is clear that $\Delta V_{2,j,k}=\int_0^T\lambda_{2,j,k}\gamma_{2,j,k}\dot{\gamma}_{2,j,k}d\tau$.

For $\dot{\gamma}_{2,j,k}$, we can derive that $\dot{\gamma}_{2,j,k}=(\varepsilon_j+d_j)\dot{x}_{2,j,k}-\dot{\sigma}_{1,j,k}=\gamma_{3,j,k}-\lambda_{2,j,k}^{-1}\mu_{2,j}\gamma_{2,j,k}-\lambda_{2,j,k}^{-1}\lambda_{1,j,k}\gamma_{1,j,k}$. We have $\Delta V_{2,j,k}=\int_0^T(\lambda_{2,j,k}\gamma_{2,j,k}\gamma_{3,j,k}-\mu_{2,j}\gamma_{2,j,k}^2-\lambda_{1,j,k}\gamma_{1,j,k}\gamma_{2,j,k})d\tau$. Thus, we come to

$$\Delta V_{1,j,k}+\Delta V_{2,j,k}=\int_0^T\left(\lambda_{2,j,k}\gamma_{2,j,k}\gamma_{3,j,k}-\sum_{i=1}^{2}\mu_{i,j}\gamma_{i,j,k}^2\right)d\tau.$$

Indeed, we always have

$$\begin{aligned}\dot{\gamma}_{i,j,k}&=(\varepsilon_j+d_j)\dot{x}_{i,j,k}-\dot{\sigma}_{i-1,j,k}\\&=\gamma_{i+1,j,k}-\lambda_{i,j,k}^{-1}\mu_{i,j}\gamma_{i,j,k}-\lambda_{i,j,k}^{-1}\lambda_{i-1,j,k}\gamma_{i-1,j,k},\end{aligned}$$

for $i=2,3,\dots,n-1$. Therefore, by mathematical induction principle, we can show that

$$\sum_{i=1}^{n-1}\Delta V_{i,j,k}(T)=\int_0^T\left(\lambda_{n-1,j,k}\gamma_{n-1,j,k}\gamma_{n,j,k}-\sum_{i=1}^{n-1}\mu_{i,j}\gamma_{i,j,k}^2\right)d\tau. \tag{17}$$

For the last term of $\Delta V_{j,k}^1(T)$, i.e., $\Delta V_{n,j,k}(T)$, we have

$$\Delta V_{n,j,k}(T)=\int_0^T\lambda_{n,j,k}\gamma_{n,j,k}\dot{\gamma}_{n,j,k}d\tau, \tag{18}$$

where $\dot{\gamma}_{n,j,k}=(\varepsilon_j+d_j)\dot{x}_{n,j,k}-\dot{\sigma}_{n-1,j,k}=-(\varepsilon_j+d_j)\tilde{\theta}_{j,k}^T\xi_{j,k}+(\varepsilon_j+d_j)\hat{\theta}_{j,k}^T\xi_{j,k}+(\varepsilon_j+d_j)b_{j,k}u_{j,k}+\eta_{j,k}-\lambda_{n,j,k}^{-1}\mu_{n,j}\gamma_{n,j,k}-\lambda_{n,j,k}^{-1}\lambda_{n-1,j,k}\gamma_{n-1,j,k}-\sigma_{n,j,k}$ with $\tilde{\theta}_{j,k}\triangleq\hat{\theta}_{j,k}-\theta_j$. Substituting (6) into this equation and noticing the basic inequality $m_k\le\frac{b_{j,k}}{b_{\min}}m_k\mathrm{sgn}(m_k)$, where m_k denotes $\lambda_{n,j,k}\gamma_{n,j,k}\hat{\theta}_{j,k}^T\xi_{j,k}$,

$-\lambda_{n,j,k}\gamma_{n,j,k}\sigma_{n,j,k}$, and $\lambda_{n,j,k}\gamma_{n,j,k}\eta_{j,k}$, respectively, we have

$$\begin{aligned}\Delta V_{n,j,k}(T) \le \int_0^T \Big[&- \lambda_{n,j,k}\gamma_{n,j,k}(\varepsilon_j + d_j)\tilde{\theta}_{j,k}^T \xi_{j,k} \\ &+ \lambda_{n,j,k}\gamma_{n,j,k}(\varepsilon_j + d_j) b_{j,k}\hat{u}_{j,k} \\ &- \mu_{n,j}\gamma_{n,j,k}^2 - \lambda_{n-1,j,k}\gamma_{n-1,j,k}\gamma_{n,j,k}\Big] d\tau, \end{aligned} \tag{19}$$

which, combining with (17), further yields that

$$\begin{aligned}\Delta V_{j,k}^1(T) = \int_0^T \Big[&-\lambda_{n,j,k}\gamma_{n,j,k}(\varepsilon_j + d_j)\tilde{\theta}_{j,k}^T \xi_{j,k} \\ &+ \lambda_{n,j,k}\gamma_{n,j,k}(\varepsilon_j + d_j) b_{j,k}\hat{u}_{j,k} - \sum_{i=1}^n \mu_{i,j}\gamma_{i,j,k}^2\Big] d\tau. \end{aligned} \tag{20}$$

Next, we proceed to the term $V_{j,k}^2(T)$.

$$\begin{aligned}\Delta V_{j,k}^2(T) =& \frac{\varepsilon_j + d_j}{2p_j} \int_0^T (\hat{\theta}_{j,k} - \hat{\theta}_{j,k-1})^T (\hat{\theta}_{j,k} + \hat{\theta}_{j,k-1} - 2\theta_j) d\tau \\ \le& \frac{\varepsilon_j + d_j}{p_j} \int_0^T (\hat{\theta}_{j,k} - \theta_j)^T (\hat{\theta}_{j,k} - \hat{\theta}_{j,k-1}) d\tau \\ =& (\varepsilon_j + d_j) \int_0^T \lambda_{n,j,k}\gamma_{n,j,k}\tilde{\theta}_{j,k}^T \xi_{j,k} d\tau, \end{aligned} \tag{21}$$

where (8) is used for the last equality.

Then, for the last term $\Delta V_{j,k}^3(T)$, we have

$$\begin{aligned}\Delta V_{j,k}^3(T) =& \frac{\varepsilon_j + d_j}{2q_j} b_{j,k} \left[\int_0^T \hat{u}_{j,k}^2 d\tau - \int_0^T \hat{u}_{j,k-1}^2 d\tau \right] \\ \le& \frac{\varepsilon_j + d_j}{q_j} b_{j,k} \int_0^T \hat{u}_{j,k}(\hat{u}_{j,k} - \hat{u}_{j,k-1}) d\tau \\ =& -(\varepsilon_j + d_j) b_{j,k} \int_0^T \lambda_{n,j,k}\gamma_{n,j,k}\hat{u}_{j,k} d\tau, \end{aligned} \tag{22}$$

where (7) is used in the last equality.

Consequently, combining (20)–(22) results in $\Delta E_{j,k}(T) \le -\int_0^T \left(\sum_{i=1}^n \mu_{i,j}\gamma_{i,j,k}^2\right) d\tau$, which further yields

$$\Delta E_k(T) = \sum_{j=1}^N \Delta E_{j,k}(T) \le -\sum_{j=1}^N \int_0^T \left(\sum_{i=1}^n \mu_{i,j}\gamma_{i,j,k}^2\right) d\tau. \tag{23}$$

Thus the decreasing property of BCEF in iteration domain at $t = T$ is obtained.

Part II. Finiteness of $E_k(t)$ and involved quantities

The finiteness of $E_k(t)$ will be proved for the first iteration and then generalized to the following iterations. To this end, we first give the expressions of $\dot{E}_k(t)$ and then show the finiteness of $E_1(t)$. For any k, we have $\dot{E}_k(t) = \sum_{j=1}^N \dot{E}_{j,k}(t) = \sum_{j=1}^N (\dot{V}_{j,k}^1(t) + \dot{V}_{j,k}^2(t) + \dot{V}_{j,k}^3(t))$.

Similar to the derivations in Part I, for $\dot{V}_{j,k}^1(t)$, we have

$$\begin{aligned}\dot{V}_{j,k}^1(t) \le& -\lambda_{n,j,k}\gamma_{n,j,k}(\varepsilon_j + d_j)\tilde{\theta}_{j,k}^T \xi_{j,k} \\ &+ \lambda_{n,j,k}\gamma_{n,j,k}(\varepsilon_j + d_j) b_{j,k}\hat{u}_{j,k} - \sum_{i=1}^n \mu_{i,j}\gamma_{i,j,k}^2. \end{aligned} \tag{24}$$

For $\dot{V}_{j,k}^2(t)$, we have

$$\begin{aligned}&\frac{2p_j}{\varepsilon_j + d_j}\dot{V}_{j,k}^2(t) = (\hat{\theta}_{j,k} - \theta_j)^T (\hat{\theta}_{j,k} - \theta_j) \\ \le& \theta_j^T\theta_j - 2\theta_j^T\hat{\theta}_{j,k-1} + \hat{\theta}_{j,k-1}^T\hat{\theta}_{j,k-1} \\ &- 2p_j\lambda_{n,j,k}\gamma_{n,j,k}\theta_j^T\xi_{j,k} + 2p_j\hat{\theta}_{j,k}^T\lambda_{n,j,k}\gamma_{n,j,k}\xi_{j,k} \\ =& (\hat{\theta}_{j,k-1} - \theta_j)^T (\hat{\theta}_{j,k-1} - \theta_j) + 2p_j\tilde{\theta}_{j,k}^T\lambda_{n,j,k}\gamma_{n,j,k}\xi_{j,k}. \end{aligned}$$

Further, for $\dot{V}_{j,k}^3(t)$, we have

$$\begin{aligned}\frac{2q_j}{(\varepsilon_j + d_j)b_{j,k}}\dot{V}_{j,k}^3(t) =& \hat{u}_{j,k}^2 = (\hat{u}_{j,k-1} - q_j\lambda_{n,j,k}\gamma_{n,j,k})^2 \\ \le& \hat{u}_{j,k-1}^2 - 2q_j\hat{u}_{j,k}\lambda_{n,j,k}\gamma_{n,j,k}. \end{aligned}$$

Combining the above three inequalities of $\dot{V}_{j,k}^1$, $\dot{V}_{j,k}^2$, and $\dot{V}_{j,k}^3$ together leads to

$$\begin{aligned}\dot{E}_{j,k} \le& -\sum_{i=1}^n \mu_{i,j}\gamma_{i,j,k}^2 + \frac{\varepsilon_j + d_j}{2q_j} b_{j,k}\hat{u}_{j,k-1}^2 \\ &+ \frac{\varepsilon_j + d_j}{2p_j}(\hat{\theta}_{j,k-1} - \theta_j)^T (\hat{\theta}_{j,k-1} - \theta_j). \end{aligned} \tag{25}$$

It can be derived from (23) that the finiteness or boundedness of $E_k(T)$ is ensured for each iteration provided that $E_1(T)$ is finite. Thus, we now verify the finiteness of $E_1(t)$.

It is found from (25) $\dot{E}_{j,1} \le -\sum_{i=1}^n \mu_{i,j}\gamma_{i,j,1}^2 + \frac{\varepsilon_j + d_j}{2p_j}\theta_j^T\theta_j$, because the initial values of (7) and (8) are set to be zero, i.e., $\hat{u}_{j,0} = 0$ and $\hat{\theta}_{j,0} = 0$. Clearly, $\dot{E}_{j,1}(t)$ is bounded over $[0, T]$, $\forall j$. Hence, the boundedness of $E_{j,1}(t)$ over $[0, T]$ is also obtained, $\forall j$. In particular, $E_{j,1}(T)$ is bounded. Noticing $E_1(T) = \sum_{j=1}^N E_{j,1}(T)$, we have that $E_1(T)$ is bounded.

Now, we are in the position of checking the boundedness property of $E_k(t)$ for $k \ge 2$, the parameter estimation $\hat{\theta}_{j,k}$ and the control signal $\hat{u}_{j,k}$. According to the definition of $E_k(t)$ and the boundedness of $E_k(T)$, the boundedness of $V_{j,k}^2(T)$ and $V_{j,k}^3(T)$ are guaranteed for all iterations. That is, for any $k \in \mathbb{Z}^+$, there exist constants $M_1 > 0$ and $M_2 > 0$ such that $\int_0^t (\hat{\theta}_{j,k} - \theta_j)^T (\hat{\theta}_{j,k} - \theta_j) d\tau \le \int_0^T (\hat{\theta}_{j,k} - \theta_j)^T (\hat{\theta}_{j,k} - \theta_j) d\tau \le M_1 < \infty$, $\int_0^t b_{j,k}\hat{u}_{j,k}^2 d\tau \le \int_0^T b_{j,k}\hat{u}_{j,k}^2 d\tau \le M_2 < \infty$. Hence, the boundedness of $\hat{\theta}_{j,k}$ and $\hat{u}_{j,k}$ is guaranteed.

Recalling the differential of $E_{j,k}$ in (25), we have

$$\begin{aligned}E_{j,k}(t) =& E_{j,k}(0) + \int_0^t \Big(-\sum_{i=1}^n \mu_{i,j}\gamma_{i,j,k}^2 + \frac{\varepsilon_j + d_j}{2q_j} b_{j,k}\hat{u}_{j,k-1}^2 \\ &+ \frac{\varepsilon_j + d_j}{2p_j}(\hat{\theta}_{j,k-1} - \theta_j)^T (\hat{\theta}_{j,k-1} - \theta_j)\Big) d\tau \\ \le& E_{j,k}(0) + \frac{\varepsilon_j + d_j}{2p_j} M_1 + \frac{\varepsilon_j + d_j}{2q_j} M_2. \end{aligned}$$

Meanwhile, $E_{j,k}(0) = E_{j,k-1}(T)$ is also bounded by the alignment condition. Therefore, it is evident that $E_{j,k}(t)$ is bounded over $[0, T]$. And so is the amount $E_k(t)$.

Part III. Convergence of extended observation errors

We recall that $\Delta E_k(T) \le -\sum_{j=1}^N \int_0^T \sum_{i=1}^n \mu_{i,j}\gamma_{i,j,k}^2 d\tau$. Thus, $E_k(T) \le E_1(T) - \sum_{l=2}^k \sum_{j=1}^N \int_0^T \sum_{i=1}^n \mu_{i,j}\gamma_{i,j,k}^2 d\tau$. As $E_k(T)$ is positive and $E_1(T)$ is bounded, $\sum_{l=2}^k \sum_{j=1}^N \int_0^T \sum_{i=1}^n \mu_{i,j}\gamma_{i,j,k}^2 d\tau$ is bounded $\forall k$. Then, $\gamma_{i,j,k}$ converges to zero asymptotically in the sense of L_T^2-norm, i.e., $\lim_{k\to\infty} \int_0^T \gamma_{i,j,k}^2 d\tau = 0$, $\forall i, j$. Moreover, as $\gamma_{1,j,k} = z_{1,j,k}$, we have actually obtained the convergence of extended observation errors $\bar{z}_{1,k}$ in the sense of L_T^2-norm, i.e., $\lim_{k\to\infty} \int_0^T \|\bar{z}_{1,k}\|_2^2 d\tau = 0$. Further, consider the convergence of the second dimension of extended observation error $\bar{z}_{2,k}$. Because $\gamma_{1,j,k} \to 0$, we have $\sigma_{1,j,k} \to \varepsilon_j\dot{x}_{1,r} + \sum_{l=1}^N a_{jl}\dot{x}_{1,l,k} = \varepsilon_j x_{2,r} + \sum_{l=1}^N a_{jl}x_{2,l,k}$ and then $\gamma_{2,j,k} \to z_{2,j,k}$ in the sense of L_T^2-norm. As a result, we have $\lim_{k\to\infty} \int_0^T \|\bar{z}_{2,k}\|_2^2 d\tau = 0$. By mathematical induction principle, one can show $\lim_{k\to\infty} \int_0^T \|\bar{z}_{i,k}\|_2^2 d\tau = 0$, $i = 3, \ldots, n$ similarly.

Part IV. Constraints verification on states

In the last part, we have shown that $E_k(t)$ is bounded over $[0, T]$ for all iterations. So it is guaranteed that $V_{i,j,k}(t)$, i.e., $V(\gamma_{i,j,k}^2(t), t)$, is bounded over $[0, T]$ for all dimensions, all iterations and all agents. According to the definition of the so-called γ-type BLF, we can conclude that $|\gamma_{i,j,k}| < k_{b_{i,j}}$ holds over $[0,T]$, $\forall i = 1,\dots,n$, $\forall j = 1,\dots,N$, $\forall k \in \mathbb{Z}^+$. Noticing that $\gamma_{1,j,k} = z_{1,j,k}$, we have $|z_{1,j,k}| < k_{b_{1,j}}$, $\forall j = 1,\dots,N$. Denote $k_{1,m} \triangleq \max_j k_{b_{1,j}}$. Clearly, $\|\bar{z}_{1,k}\| \le N k_{1,m}$, $\forall k \in \mathbb{Z}^+$.

On the other hand, the relationship between $\bar{z}_{1,k}$ and $\bar{e}_{1,k}$ in (2) leads to $\bar{e}_{1,k} = \mathscr{H}^{-1}\bar{z}_{1,k}$. This further yields $\|\bar{e}_{1,k}\| \le \sigma_{\max}(\mathscr{H}^{-1})\|\bar{z}_{1,k}\| \le \frac{1}{\sigma_{\min}(\mathscr{H})} N k_{1,m}$ for all $k \in \mathbb{Z}^+$.

For the constraints imposed on $x_{1,j,k}$, $|x_{1,j,k}| < k_{s,1}$, we can set $k_{1,m} = (k_{s,1} - |x_{1,r}|)\sigma_{\min}(\mathscr{H})/N$. Under this setting, the tracking error will be bounded as follows: $|e_{1,j,k}| \le \|\bar{e}_{1,k}\| \le \frac{1}{\sigma_{\min}(\mathscr{H})} N k_{1,m} \le \frac{1}{\sigma_{\min}(\mathscr{H})} N (k_{s,1} - |x_{1,r}|)\frac{\sigma_{\min}(\mathscr{H})}{N} = k_{s,1} - |x_{1,r}|$ In other words, $|x_{1,j,k} - x_{1,r}| = |e_{1,j,k}| \le k_{s,1} - |x_{1,r}|$, and therefore $|x_{1,j,k}| \le k_{s,1} - |x_{1,r}| + |x_{1,r}| = k_{s,1}$. The constraint of the first-dimension of the state is satisfied.

Now, for the constraint on $x_{2,j,k}$, we can define $\bar{\gamma}_{2,k} = [\gamma_{2,1,k},\dots,\gamma_{2,N,k}]^T$ and $\varphi = [\frac{\mu_{1,1}\gamma_{1,1,k}}{\lambda_{1,1,k}},\dots,\frac{\mu_{1,N}\gamma_{1,N,k}}{\lambda_{1,N,k}}]^T$. Then, we have $\bar{\gamma}_{2,k} = \mathscr{H}\bar{e}_{2,k} + \varphi$, or equivalently, $\|\bar{e}_{2,k}\| \le \frac{1}{\sigma_{\min}(\mathscr{H})}(\|\bar{\gamma}_{2,k}\| + \|\varphi\|)$. Therefore, to ensure the constraint, it suffices to satisfy $\frac{1}{\sigma_{\min}(\mathscr{H})}(\|\bar{\gamma}_{2,k}\| + \|\varphi\|) \le k_{s,2} - |x_{2,r}|$. This is valid if $k_{2,m} \le \frac{1}{N}[\sigma_{\min}(\mathscr{H})(k_{s,2} - |x_{2,r}|) - \|\varphi\|]$ is satisfied, where $k_{2,m} \triangleq \max_j k_{b_{2,j}}$.

In addition, the unknown function $\xi_{j,k}$ is bounded as its argument $x_{i,j,k}$ has been shown to be bounded. Incorporating with the result that $\gamma_{i,j,k}$ and $\lambda_{i,j,k}$ are bounded, and noting (6), we can conclude that the input profile $u_{j,k}$ is also bounded.

Part V. Uniformly consensus tracking

In the last part, it is shown that $|\gamma_{i,j,k}|$ is bounded by $k_{b_{i,j}}$ for all iterations. Recall that $\gamma_{i,j,k}$ also converges to zero in the sense of L_T^2-norm, as is shown in Part IV. Then we can conclude that $\gamma_{i,j,k} \to 0$ uniformly as $k \to \infty$, $\forall i, j$. In other words, $z_{i,j,k} \to 0$ uniformly as $k \to \infty$, $\forall i, j$. Then $\bar{z}_{i,k} \to 0$. Meanwhile, $\bar{z}_{i,k} = \mathscr{H}\bar{e}_{i,k}$ and $\mathscr{H}$ is an invertible matrix. Thus, $\bar{e}_{i,k} \to 0$ uniformly as $k \to \infty$. In short, the uniform consensus tracking is proved. The proof is completed. □

Proof of Theorem 2. We still apply the BCEF given in (13)–(16) and check the difference of $E_k(T)$ first.

Part I. Difference of $E_k(t)$

The steps from beginning to (18) of proof of Theorem 1 are still valid, and thus are not copied here. Now substitute (9) into the expression of $\dot{\gamma}_{n,j,k}$. Using Lemma 1, we can substitute the terms $\lambda_{n,j,k}\gamma_{n,j,k}\hat{\theta}_{j,k}^T\xi_{j,k}$, $\lambda_{n,j,k}\gamma_{n,j,k}\sigma_{n,j,k}$, and $\lambda_{n,j,k}\gamma_{n,j,k}\eta_{j,k}$ as χ and then we obtain the estimate of the difference terms (similar to the derivations for (19)). Let $\bar{\delta}$ be a constant satisfying $\bar{\delta} > \frac{b_{j,k}}{b_{\min}}(\varepsilon_j + d_j)\delta$. Then, we have

$$\sum_{i=1}^{n}\Delta V_{i,j,k}(T) = \int_0^T \Big[-\lambda_{n,j,k}\gamma_{n,j,k}(\varepsilon_j + d_j)\tilde{\theta}_{j,k}^T\xi_{j,k} + \lambda_{n,j,k}\gamma_{n,j,k}(\varepsilon_j + d_j)b_{j,k}\hat{u}_{j,k}\Big]d\tau - \int_0^T\Big(\sum_{i=1}^{n}\mu_{i,j}\gamma_{i,j,k}^2\Big)d\tau + 3T\bar{\delta}\varepsilon. \quad (26)$$

Combining with (21) and (22) we further have $\Delta E_{j,k}(T) \le -\int_0^T\left(\sum_{i=1}^{n}\mu_{i,j}\gamma_{i,j,k}^2\right)d\tau + 3T\bar{\delta}\varepsilon$ and

$$\Delta E_k(T) \le -\sum_{j=1}^{N}\int_0^T\Big(\sum_{i=1}^{n}\mu_{i,j}\gamma_{i,j,k}^2\Big)d\tau + 3NT\bar{\delta}\varepsilon. \quad (27)$$

Part II. Bounded convergence analysis

In the last part, the difference of $E_k(T)$ is obtained, i.e., $\Delta E_k(T) \le -\mu_m\int_0^T\left(\sum_{j=1}^{N}\sum_{i=1}^{n}\gamma_{i,j,k}^2\right)d\tau + 3NT\bar{\delta}\varepsilon$, where $\mu_m \triangleq \min_{i,j}\mu_{i,j}$. Then finite summation of $\Delta E_k(T)$ from the first iteration leads to

$$E_k(T) = E_1(T) + \sum_{l=1}^{k}\Delta E_k(T) \le E_1(T) - \mu_m\sum_{l=1}^{k}\Big[\int_0^T\Big(\sum_{j=1}^{N}\sum_{i=1}^{n}\gamma_{i,j,k}^2\Big)d\tau - \frac{3NT\bar{\delta}\varepsilon}{\mu_m}\Big]. \quad (28)$$

Due to the positiveness of $E_k(T)$, we can show the boundedness and convergence of $\gamma_{i,j,k}$ from (28).

(a) If $\int_0^T\left(\sum_{j=1}^{N}\sum_{i=1}^{n}\gamma_{i,j,k}^2\right)d\tau$ goes to infinity at the kth iteration, then the right hand side (RHS) of (28) will diverge to infinity owing to the finiteness of $3NT\bar{\delta}\varepsilon/\mu_m$. This contradicts the positiveness of $E_k(T)$.

(b) For any given $\nu > 0$, there is a finite integer $k_0 > 0$ such that $\int_0^T\left(\sum_{j=1}^{N}\sum_{i=1}^{n}\gamma_{i,j,k}^2\right)d\tau < 3NT\bar{\delta}\varepsilon/\mu_m + \nu$ for $k \ge k_0$. Otherwise, $\int_0^T\left(\sum_{j=1}^{N}\sum_{i=1}^{n}\gamma_{i,j,k}^2\right)d\tau \ge 3NT\bar{\delta}\varepsilon/\mu_m + \nu$ holds for $k \to \infty$. Then the RHS of (28) will approach $-\infty$, which again contradicts the positiveness of $E_k(T)$.

Hence, the summation of L_T^2-norm of fictitious errors will enter the specified bound $3NT\bar{\delta}\varepsilon/\mu_m + \nu$ within finite iterations.

Next we transfer the above convergence to the extended observation error and tracking error. To this end, denote $\gamma_{j,k} \triangleq [\gamma_{1,j,k},\dots,\gamma_{n,j,k}]^T$. From the definition of fictitious errors and stabilizing functions, we have $z_{j,k} = \Gamma_{j,k}\gamma_{j,k}$, where $\Gamma_{j,k}$ is defined as

$$\Gamma_{j,k} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -\lambda_{1,j,k}^{-1}\mu_{1,j} & 1 & 0 & \cdots & 0 \\ -\lambda_{2,j,k}^{-1}\lambda_{1,j,k} & -\lambda_{2,j,k}^{-1}\mu_{2,j} & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & 1 \end{bmatrix}.$$

According to Definition 1, it is seen $\lambda_{i,j,k}^{-1}$ is bounded. Therefore, any matrix norm of $\Gamma_{j,k}$ is bounded for any j and k. For clarity, we assume $\kappa > \sigma_{\max}(\Gamma_{j,k})$, $\forall j, k$. Then we have $\sum_{i=1}^{n}\int_0^T z_{i,j,k}^2 d\tau = \int_0^T\|z_{j,k}\|^2 d\tau \le \kappa^2\int_0^T\|\gamma_{j,k}\|^2 d\tau$. It further leads to $\sum_{j=1}^{N}\sum_{i=1}^{n}\int_0^T z_{i,j,k}^2 d\tau \le \kappa^2\sum_{j=1}^{N}\sum_{i=1}^{n}\int_0^T\gamma_{i,j,k}^2 d\tau$. Consequently, the summation of L_T^2-norm of extended observation errors would converge to the ζ_z-neighborhood of zero within finite iterations, where $\zeta_z = \frac{3\kappa^2 NT\bar{\delta}\varepsilon}{\mu_m} + \kappa^2\nu$.

From (2), we have $\bar{e}_{i,k} = \mathscr{H}^{-1}\bar{z}_{i,k}$, $\forall 1 \le i \le n, k$. This hints that $\int_0^T\|\bar{e}_{i,k}\|^2 d\tau \le \frac{1}{\sigma_{\min}^2(\mathscr{H})}\int_0^T\|\bar{z}_{i,k}\|^2 d\tau$, which further yields $\sum_{j=1}^{N}\sum_{i=1}^{n}\int_0^T e_{i,j,k}^2 d\tau \le \frac{1}{\sigma_{\min}^2(\mathscr{H})}\sum_{j=1}^{N}\sum_{i=1}^{n}\int_0^T z_{i,j,k}^2 d\tau$. Then, the summation of L_T^2-norm of tracking errors converges to the ζ_e-neighborhood of zero within finite iterations, where $\zeta_e = \frac{3\kappa^2 NT\bar{\delta}\varepsilon}{\sigma_{\min}^2(\mathscr{H})\mu_m} + \frac{\kappa^2\nu}{\sigma_{\min}^2(\mathscr{H})}$. The proof is completed. □

Proof of Corollary 1. First, similar to the proof of Theorem 2, we can select ε and ν sufficiently small such that $\zeta_z < \varsigma$ for any prior given ς. Thus, the finite iteration convergence to a predefined neighborhood of zero holds by the proof of Theorem 2. Moreover, once the tracking error enters the predefined neighborhood, the learning processes (11)–(12) stop updating and the boundedness is therefore guaranteed evidently.

Next, let us verify the output constraint satisfaction. Note that the tracking error would enter a neighborhood of zero within finite iterations and the neighborhood magnitude can be predefined. Therefore, for a given neighborhood bound, a finite integer exists, say k_1, such that the tracking error enters the given neighborhood for $k \geq k_1$. It is evident that $E_k(T)$ is bounded, $\forall k < k_1$. Thus $V^1_{j,k}$ is also bounded, $\forall k < k_1$, whence the constraints can be verified similarly to the proof of Theorem 1. When $k \geq k_1$, the tracking error will enter a predefined neighborhood. Then, the learning processes (11)–(12) would stop updating and the control system would repeat its tracking performance. Consequently, the output constraints are still valid. This completes the proof. □

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